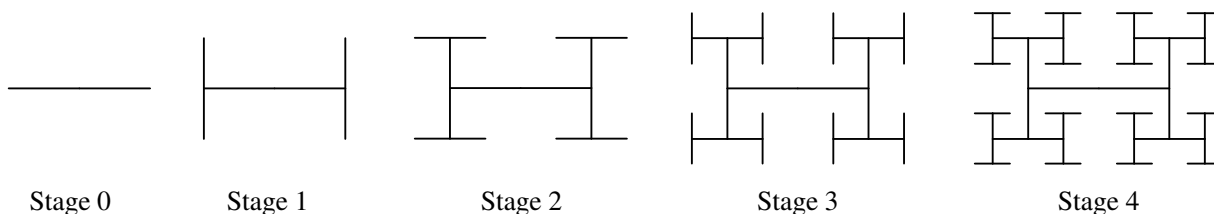


H TREE

COURSE/LEVEL: NSW Secondary High School Stage 5 Mathematics – Additional Content

Starting with a line segment, perpendicular branches are attached at their midpoints to both ends. The ratio of these two branches to the original line segment is less than one. At each stage of iteration, branches are attached in the same way to the existing endpoints. Successive branches decrease in length by the same ratio and this process is continued indefinitely. (Note that the H Tree is merely a fractal tree (minus the “stem”), where the angle between branches is $\pm 90^\circ$.)



In the H Tree drawn above, successive branches decrease in length by the factor $\frac{\sqrt{2}}{2}$.

1. Complete the table below.

Stage	Number of new branches	Total number of branches	Length of each new branch	Total length of new branches
0	1	1	1	1
1	2	3	$\frac{\sqrt{2}}{2}$	$\sqrt{2}$
2	4	7	$\frac{1}{2}$	2
3	8	15	$\frac{\sqrt{2}}{4}$	$2\sqrt{2}$
4	16	31	$\frac{1}{4}$	4
5				
6				
n				

2. Complete the following table.

	Stage 0	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5	Stage 6
Increase in width of tree	1	0	$\frac{1}{2}$	0	$\frac{1}{4}$		

3. If w is the width of the H tree, show that $w = 1 + \frac{1}{2} + \frac{1}{4} + \dots$
4. Use the formula $1 + r + r^2 + \dots = \frac{1}{1-r}$ (where $-1 < r < 1$), to show that the width of the H Tree is 2.
5. Explain why the height of the H Tree is $\sqrt{2}$.