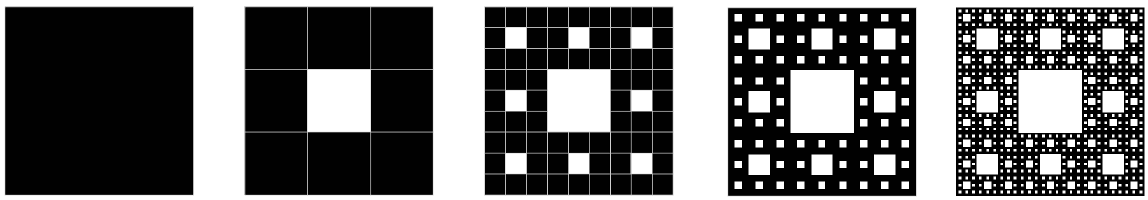
SIERPINSKI CARPET

COURSE/LEVEL: *NSW Secondary High School Stage 5 Mathematics – Additional Content*

In creating the Sierpinski Carpet, a square is firstly divided into nine smaller squares and the central square is removed. Each of the eight remaining squares are then divided into nine smaller squares and their central squares are removed. This process continues indefinitely.



Stage 0 Stage 1 Stage 2 Stage 3 Stage 4

1. At the *n*th stage of iteration of the Sierpinski carpet, let B_n equal the area of remaining (black) squares. Explain why $B_n = \frac{8}{9} \times B_{n-1}$ for $n = 1, 2, \ldots$, and complete the table below.

	Stage 0	Stage 1	Stage 2	Stage 3	Stage *n*
Number of black squares	1	8	8^2		
Area of black squares	A	$\frac{8}{9} \times A$	$\left(\frac{8}{9}\right)^2 \times A$		

2. Comment on the number of remaining (black) squares and their total area as the number of iterations, *n*, approaches infinity.

3. Complete the table below.

Stage n	Number of *new* white squares	Total number of white squares	Number of black squares	Total number of squares
0	0	0	1	1
1	1	1	8	1 + 8
2	8	1 + 8	8^2	$1+8+8^2$
3				
4				
5				

4. Use the formula $1 + r + r^2 + r^3 + \ldots + r^n = \frac{r^{n+1} - 1}{r - 1}$ to find an expression for the total number of squares at the *n*th stage of iteration.