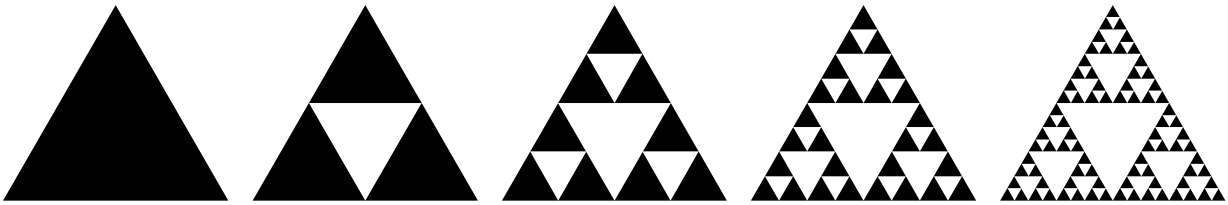


SIERPINSKI TRIANGLE

COURSE/LEVEL: *NSW Secondary High School Stage 5 Mathematics – Additional Content*

Beginning with an equilateral triangle, the central equilateral triangle with vertices at the midpoints of each side is removed. At each stage of iteration, the central triangle is removed from each of the remaining triangles.



Stage 0 Stage 1 Stage 2 Stage 3 Stage 4

1. Complete the table below.

	Stage 0	Stage 1	Stage 2	Stage 3	Stage n
Number of black triangles	1	3	3^2		
Area of black triangles	A	$\frac{3}{4}\times A$	$\left(\frac{3}{4}\right)^2\times A$		
Perimeter of black triangles	P	$\frac{3}{2}\times P$	$\left(\frac{3}{2}\right)^2\times P$		

2. Comment on the area and perimeter of black triangles as the number of iterations, n, approaches infinity.

3. Complete the table below.

Stage n	Number of *new* white triangles	Total number of white triangles	Number of black triangles	Total number of triangles
0	0	0	1	1
1	1	1	3	1 + 3
2	3	1 + 3	3^2	$1 + 3 + 3^2$
3				
4				
5				

4. Use the formula $1+r+r^2+r^3+\ldots+r^n=\frac{r^{n+1}-1}{r-1}$ to find an expression for the total number of triangles at the nth stage of iteration.