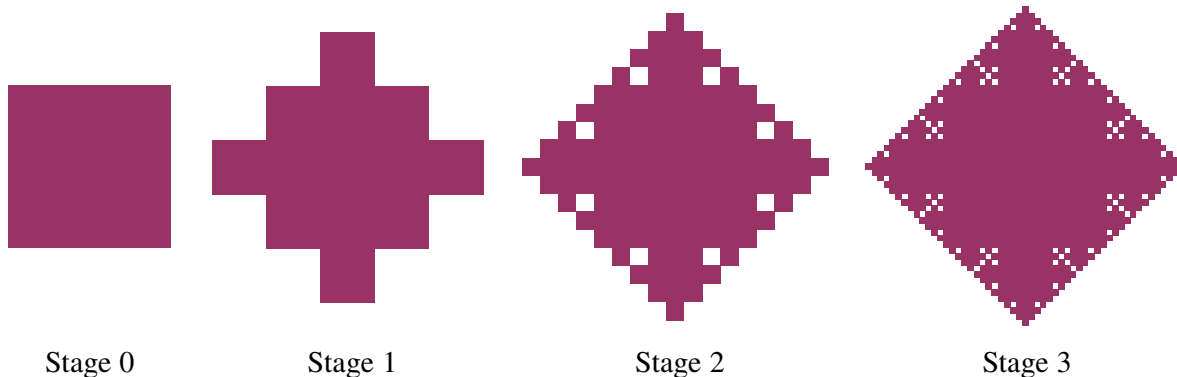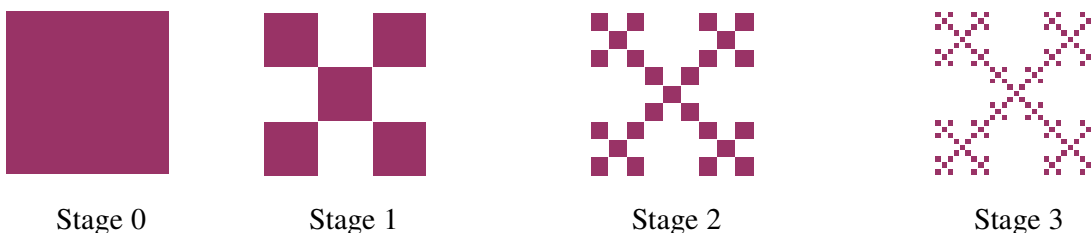# SQUARE SNOWFLAKE/ANTI-SNOWFLAKE

COURSE/LEVEL: *NSW Secondary High School Stage 5 Mathematics – Additional Content*

The square snowflake is created in a very similar way to the Koch snowflake. Starting with a square, smaller squares are added to the middle third of each side at each iteration. The first few stages of iteration are shown below.

| Stage 0 | Stage 1 | Stage 2 | Stage 3 |

The Anti-Snowflake is produced in much the same way, except squares are removed from the middle third of each side in each iteration.

| Stage 0 | Stage 1 | Stage 2 | Stage 3 |

1. (a) Consider the square snowflake at the nth stage of iteration and its corresponding anti-snowflake at the same stage of iteration.

   Explain why

   (i) The two shapes are equal in perimeter.
   (ii) The sum total area of the two shapes is equal to $2A_0$, that is twice the area of the original square at Stage 0.

2. At the nth stage of iteration of the Square Anti-snowflake, I_n equals the number of sides, L_n equals the side length, P_n is its perimeter and A_n is its area.

   (a) Explain the following, for $n = 1, 2, \ldots$,

   (i) $I_n = 5 \times I_{n-1}$ (ii) $L_n = \dfrac{L_{n-1}}{3}$ (iii) $P_n = \dfrac{5}{3} \times P_{n-1}$ (iv) $A_n = \dfrac{5}{9} \times A_{n-1}$

   (b) Use these results to complete the table below. (Let l be the side length of the square at Stage 0).

Stage	Number of Sides	Side Length	Perimeter	Area
0	5	l	P_0	A_0
1	5×4	$\dfrac{l}{3}$	$5 \times P_0$	$\dfrac{5}{9} \times A_0$
2	$5^2 \times 4$	$\dfrac{l}{3^2}$	$5^2 \times P_0$	$\left(\dfrac{5}{9}\right)^2 \times A_0$
3				
4				
n				

(c)   As you increase the number of iterations, explain why

   (i)   the perimeter of the Square anti-snowflake approaches infinity.

   (ii) the area of the Square anti-snowflake approaches zero.

(d)   Hence use your answers to Question 1 to explain why the Square snowflake (where n tends to infinity) is twice the area of the square at Stage 0.