INDUCTION – WORKSHEET

COURSE/LEVEL

NSW Secondary High School Year 11 Preliminary Mathematics Extension. Syllabus reference: 7.4.

1. Prove the following by induction, where *n* is any positive integer:

(a) $1+2+3+\ldots+n=\frac{n(n+1)}{2}$.

(b) $1^2+2^2+3^2+\ldots+n^2=\frac{n(n+1)(2n+1)}{6}$.

(c) $1^3+2^3+3^3+\ldots+n^3=\frac{n^2(n+1)^2}{4}$.

(d) $1^3+2^3+3^3+\ldots+n^3=(1+2+3+\ldots n)^2$.

(e) $1+3+5+7+\ldots+(2n-1)=n^2$.

(f) $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots+\frac{1}{n(n+1)}=\frac{n}{n+1}$.

(g) $\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\ldots+\frac{1}{(3n-2)(3n+1)}=\frac{n}{3n+1}$.

(h) $1.2.3+2.3.4+3.4.5+\ldots+n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$.

2. Prove the following results for all integers $n\geq 1$.

(a) 3^n+1 is divisible by 2.

(b) $3^{2n}-1$ is divisible by 8.

(c) $5^n+2(11^n)$ is divisible by 3.

(d) $5^{2n}+5^n+2$ is divisible by 4.

3. Prove that each of the following expressions are divisible by 5 if *n* is any positive integer.

(a) $2^{3n}-3^n$.

(b) $3^{3n}+2^{n+2}$.

(c) $9^{n+2}-2^{2n}$.

(d) $13(6^n)+2$.

4. Prove the following:

(a) $3^n\geq 1+2n$.

(b) $4^n\geq 1+3n$.