PRELIMINARY MATHEMATICS EXTENSION REVISION
WORKSHEET #2

COURSE/LEVEL
NSW Secondary High School Year 11 Preliminary Mathematics Extension.

1. If $f(x) = x + 1$ and $g(x) = x - 2$, solve:

   (a) $f(x)g(x) = 0$
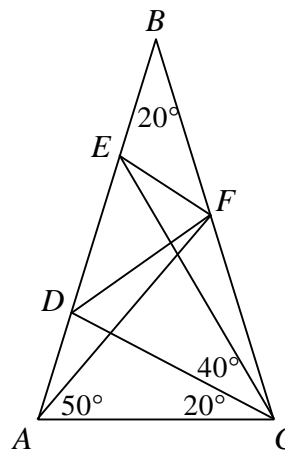
   (b) $f(x)g(x) > 0$

   (c) $\dfrac{1}{f(x)g(x)} < 0$

   (d) $\sqrt{f(x)} > \dfrac{2}{g(x) + 2}$

2. In the diagram, $\triangle ABC$ is isosceles with

   $BA = BC$, $\angle ABC = 20°$, $\angle CAF = 50°$,

 $\angle ACD = 20°$, $\angle DCE = 40°$. Find

   $\angle CEF$.

3. A car at point A is travelling due North at 100 km/h. Towns B and C are at bearings of $330°T$ and $030°T$ respectively from A. 1 hour later the car is at point D and the towns are at bearings $300°T$ and $045°T$ respectively. Find the exact distance between the two towns.

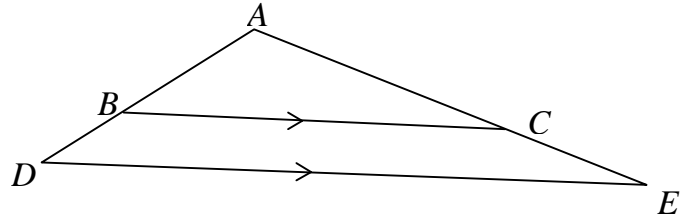4. By letting $t = \tan\dfrac{\theta}{2}$, solve the equation:

$$3\cos\theta + 4\sin\theta = 1 \text{ where } 0 \le \theta < 360°.$$

5. Find the exact value of $\tan\theta$ in the diagram if $\tan\alpha = 3$ and $\tan\beta = 2$.
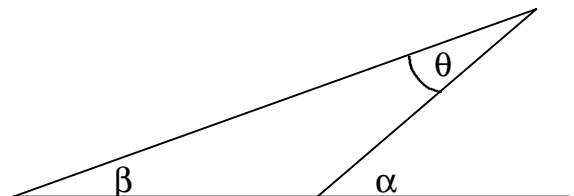
8. Solve the equation $\left(x + \dfrac{1}{x}\right)^2 - 7\left(x + \dfrac{1}{x}\right) + 12 = 0$.

6. In the diagram, $BC \parallel DE$ and $DE = \frac{5}{3}BC$. If A has coordinates (1, 5) and B has coordinates (−1, 3), find the coordinates of D.

7. The nth term of a series is given by $T_n = \dfrac{1}{(2n-1)(2n+1)}$.

    (i) Find an expression for T_{n+1}.

    (ii) Assuming that the sum S_k of the first k terms of this series is given by $S_k = \dfrac{k}{2k+1}$, prove that $S_{k+1} = \dfrac{k+1}{2k+3}$.

    (iii) Prove that $S_k = \dfrac{k}{2k+1}$ is true for all k.

9. One extremity of a focal chord of the parabola $x^2 = 4ay$ has coordinates $\left(2ap, ap^2\right)$. Find the coordinates of the focal chord's other extremity.

10. (i) Sketch the graph of $y = \dfrac{3x^2 + x - 2}{x^2 + x - 2}$, showing x and y intercepts and the equations of any vertical or horizontal asymptotes.

    (ii) Solve the equation $\dfrac{3x^2 + x - 2}{x^2 + x - 2} = 3$.

    (iii) Hence use your sketch to write down the solution to the inequality $\dfrac{3x^2 + x - 2}{x^2 + x - 2} - 3 > 0$.

11. If α, β, γ are roots of $2x^3 - 5x^2 + 3x + 2 = 0$, find:

    (i) $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$ (ii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$

12. If $x^2 - 4$ is a factor of $x^3 + 3x^2 + ax + b$, find the values of a and b.

13. The probability that a man lives to the age of 75 is $\frac{3}{5}$ and the probabilty that his sister will live to 75 is $\frac{3}{4}$. Find the probability that:

    (i) both will live to the age of 75.

    (ii) only the man will live to 75.

    (iii) at least on of them will live to 75.