PRELIMINARY MATHEMATICS EXTENSION REVISION WORKSHEET #3

COURSE/LEVEL
NSW Secondary High School Year 11 Preliminary Mathematics Extension.

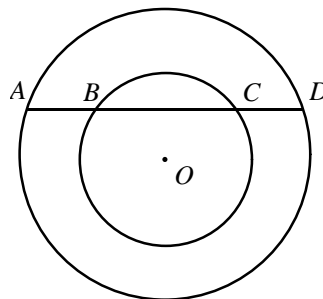**1**. Solve these inequations and graph their solutions on a numberline:

(a) $\dfrac{3}{x+5}+1\ge 0$

(b) $1+\dfrac{5}{x}+\dfrac{6}{x^2}>0$

(c) $2x<\dfrac{1}{x}$

(d) $x^4-3x^2+2>0$

2. In the diagram, O is the centre of two circles. Points A, B, C and D are collinear. Prove that $AB=CD$.


3. A surveyor sketched the diagram on the right in order to find the height of a tree across a river: the angle of elevation from point A to the treetop Q equals $6°15'$. At point B, 100 metres directly from point A towards the base of the tree at P, the angle of elevation is $8°32'$.
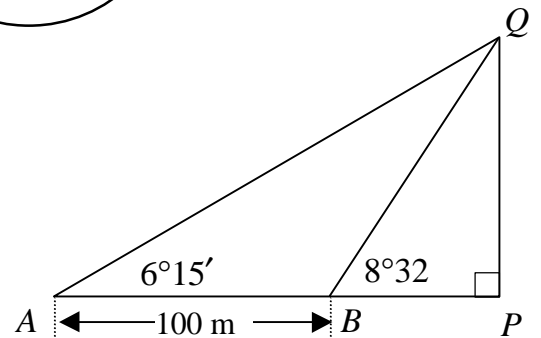
(i) Derive an expression for the exact height of the tree.

(ii) Calculate the hight of the tree in metres, correct to two decimal places.

4. (i) Show that $\dfrac{1-\cos 2x}{1+\cos 2x}=\tan^2 x$.

(ii) Hence show that the exact value of $\tan 22\tfrac{1}{2}° = \sqrt{2}-1$.

5. A straight line with positive gradient passes through the point of intersection of the two lines $y=2x-1$ and $y=-3x+4$ and bisects the angle between them. Find its equation.

6. Solve this equation for x:

$$\left(2^x+1\right)^2-7\left(2^x\right)+3=0$$

7. (i) Show that point D in the diagram divides AB in the same ratio that E divides AC.

(ii) Hence or otherwise find the coordinates of D if the coordinates of A, B, C and E are $A(0, 0)$, $B(3, 5)$, $C(3, 0)$, $E(1, 0)$.
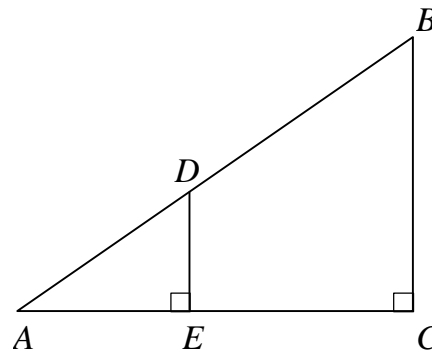
8. If n is any positive integer, prove by mathematical induction that

$$1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 = \frac{n(4n^2 - 1)}{3}.$$

9. (i) Sketch, on the one number plane graphs of $y = \sqrt{x+3}$ and $y = \dfrac{2}{x}$, showing all essential features including x and y intercepts, the equations of any horizontal or vertical asymptotes and the coordinates of all points of intersection.

(ii) Using your sketch, write down the solution to the inequation: $\sqrt{x+3} < \dfrac{2}{x}$.

10. (i) If $P(x) = x^3 - 9x^2 + 24x + k$ for some number k, find the values of x for which $P'(x) = 0$. Hence find the two values of k for which the equation $P(x) = 0$ has a multiple root.

(ii) Sketch the graphs of $y = P(x)$ for these two values of k. Hence write down the values of k for which the equation $P(x) = 0$ has three distinct roots.

11. If $x^2 - 4px + 3p - 2 = 0$:

(i) Write down an expression, involving p, for the product of the roots of the above equation.

(ii) Find the value of p given that the product of the roots is equal to three times the sum of the roots.

12. 3 books are chosen at random from a bookshelf containing 5 Mathematics textbooks and 3 Science textbooks. What is the probability of selecting:

(i) 1 Mathematics textbook only,

(ii) exactly 2 Mathematics textbooks,

(iii) at least 1 Mathematics textbook.