THE QUADRATIC POLYNOMIAL – WORKSHEET

COURSE/LEVEL

NSW Secondary High School Year 11 Preliminary Mathematics.
Syllabus reference: 9.1 – 9.4.

1. For each of the following quadratic equations determine the number of real roots by considering the value of the discriminant $\Delta = b^2 - 4ac$. If real roots exist find them using the quadratic formula.

I	**II**
$x^2 - 5x + 6 = 0$	$-x^2 + 7x - 6 = 0$
$-2x + 1 + x^2 = 0$	$4 - 12x + 9x^2 = 0$
$x^2 + x + 1 = 0$	$x - 1 + x^2 = 0$
$x^2 - 9 = 0$	$x^2 + 6x = 9$

2. Find the exact solutions of the equation $2x^2 - x - 1 = 0$.

3. (i) Sketch the graph of $y = 6 - 5x - x^2$. Make sure to show all essential features including the *x*- and *y*- intercepts and the co-ordinates of the vertex.

(ii) Use your sketch to solve the inequality $6 - 5x - x^2 \le 0$.

4. Solve these equations:

(a) $x^2 \ge (x+2)(x-3)$ (b) $(x-1)(x+1) \ge 0$

(c) $x^2 + x - 6 < 0$ (d) $12 + 4x - x^2 > 0$

5. Without solving the equation $2x^2 - 5x - 8$, determine whether the roots are:

(a) Real or unreal (b) equal or unequal (c) rational or irrational.

6. Find k if $x^2 + 3x + k = 0$ has equal roots.

7. Find the values of m if the quadratic equation $x^2 + (m-1)x - 2(m+1) = 0$ has equal roots.

8. Find the two possible values of k for which the equation $9 + kx + x^2 = 0$ has one real root.

9. Find all possible values of k for which the equation $2x^2 + 4x + k = 0$ has:

(i) two roots (ii) one root (iii) no roots

10. Find all possible values of m for which the equation $mx^2 + 2x - 1 = 0$ has:

(i) two roots (ii) one root (iii) no roots

11. Find all possible values of k for which the equation $x + \frac{x^2}{3} = k$ has:

(i) two roots (ii) one root (iii) no roots

12. Find all possible values of k for which the equation $x^2 + kx - (k+3) = 0$ has no roots.

13. Find all possible values of k for which the quadratic equation $kx^2 + (k+1)x + (2-k) = 0$ has two real roots.

14. The quadratic function $y = ax^2 + ax + a$ has all of its coefficients equal. Prove that it has no real zeros.

15. Show that the equation $x - \frac{1}{x} = k$ has two solutions for all possible values of k.

16. Consider the quadratic equation $(k+2)x^2 + (k+3)x + 1 = 0$:

(i) Show that the equation always has at least one real root for all values of k.

(ii) Find the roots if $k = \sqrt{2} - 2$.

17. Find the values of m such that the expression $x^2 + mx + 16$ is:

(i) positive definite (ii) positive definite (ii) indefinite

18. Show that if the quadratic equation $ax^2 + bx + c = 0$ has two real roots then the roots are equally spaced about the axis of symmetry and the distance between the roots is Δ.

20. If α and β are the roots of the quadratic equation $3x^2 - 6x + 2 = 0$, find the values of

(i) $\alpha + \beta$ (ii) $\alpha\beta$

(iii) $\alpha^{-1} + \beta^{-1}$ (iv) $\alpha^2 + \beta^2$

21. Find the values of k if the square of the sum of the roots of the equation $x^2 - (4-k)x + k - 3 = 0$ is equal to three times the product of the roots.

22. Find numbers a, b, c such that $a(x+1)^2 + b(x+1) + c = (2x+1)^2$ for all values of x.

22. For what values of m will the straight line with equation $y = mx + 4$

(i) touch (ii) intersect (iii) not intersect

the parabola with equation $y = 2x^2 - 2x + 5$.

23. Solve the equations: (i) $x^4 - 5x^2 + 6 = 0$ (ii) $9^x - 4.3^x + 3 = 0$.