needs to be emphasised that Newton's method is as much a geometric as it is an algebraic process, and students need to be fully aware of the graphical interpretations of the algorithm.

12.4 SIMPSON'S AND TRAPEZOIDAL RULES

There are many functions whose primitives are difficult or impossible to find. This precludes a use of calculus in the calculation of their integrals and we turn then to approximation techniques such as the Simpson's and Trapezoidal rules. Both techniques decompose the region of interest into a number of strips (sub-intervals) of equal width. The Trapezoidal rule approximates the integral by examining the area of appropriate trapezia whereas Simpson's rule groups the strips in pairs and implements approximating quadratics (parabolas). As a result Simpson's rule demands an even number of strips (and hence an odd number of function values) whereas no such restrictions apply to the Trapezoidal rule. The Trapezoidal rule offers exact solutions for linear graphs and Simpson's rule is exact for quadratics and somewhat surprisingly also for cubics.

Both rules are best executed by adopting the formal approach of treating the approximation as a weighted average of function values. The calculations for both schemes may then be streamlined into one general formula

$$A \approx (b-a)\left(\frac{\sum y_i w_i}{\sum w_i}\right)$$

where the interval of integration is [a,b], the function values are y_i and the weights w_i are chosen according to the rule to be implemented. The Trapezoidal weights take the form 122…..221 and the Simpson weights are 1424….241. The restriction that the number of strips be even, guarantees that the Simpson weights start with 14 and conclude with 41. It should be noted that the same formula applies to both techniques and that the strip width itself plays no role in the calculations. It is recommended that a table of function values together with the appropriate weights support the calculations.

It needs to be stressed that both rules are approximation techniques and should only be used where the exact machinery of the calculus is either impossible or extremely difficult to implement.

An example of both calculations follows.

Example 74 (2005 HSC Mathematics Question 6(a))

Five values of the function *f(x)* are shown in the table.

x	0	5	10	15	20
$f(x)$	15	25	22	18	10

Use Simpson's rule with the five values given in the table to estimate $\int_0^{20} f(x)dx$.

Solution

We simply augment the table with the Simpson weights.

x_i	0	5	10	15	20
y_i	15	25	22	18	10
w_i	1	4	2	4	1

Then $A \approx (b-a)\left(\dfrac{\sum y_i w_i}{\sum w_i}\right) = (20-0)\left(\dfrac{15\times 1 + 25\times 4 + 22\times 2 + 18\times 4 + 10\times 1}{1+4+2+4+1}\right) = 401\dfrac{2}{3}$

The beauty of this approach is that to use the Trapezoidal rule with 4 strips rather than Simpson's rule, we simply need to insert a modified weights row. All other calculations are unchanged!

x_i	0	5	10	15	20
y_i	15	25	22	18	10
w_i	1	2	2	2	1

The Trapezoidal calculations are then

$$A \approx (b-a)\left(\frac{\sum y_i w_i}{\sum w_i}\right) = (20-0)\left(\frac{15\times 1 + 25\times 2 + 22\times 2 + 18\times 2 + 10\times 1}{1+2+2+2+1}\right) = 387.5$$

Because of it's increased flexibility we would expect that Simpson's rule provide a better approximation to the integral than the Trapezoidal rule. The accuracy of both algorithms will in general improve with an increase in the number of strips.