2001

MATHEMATICS EXTENSION 1

HSC ASSESSMENT TASK 2

Term 1 Exams

Time allowed – Two hours

(*Plus 5 minutes reading time*)

DIRECTIONS

- Attempt ALL questions.
- EACH question is out of 12 marks.
- All necessary working should be shown. Marks may be deducted for careless or poorly arranged work.
- Board-approved calculators may be used
- Start each question on a new sheet of paper.

QUESTION 1 *Write your number on the top of the page.* **MARKS**

(a) Find the coordinates of the point that divides externally the interval joining the points $A(7, 2)$ and $B(6, 3)$ in the ratio 4:5. **3**

(b) Solve the inequality $\frac{2x+1}{x-1} > 1$. **3**

(c) Find the derivative of $\cos\left(\sin^2 x + e^x\right)$. **2**

(d) What is the focus and directrix of the parabola $x^2 = -20y$? **2**

(e) Solve $\sin 2x = \frac{-1}{2}$ for $0 \le x \le 2\pi$. **2**

QUESTION 2 *Start a new sheet of paper. Write your number on the top of the page.* **MARKS**

(a) Find $\frac{d}{dx}(\sec 3x)$. **1**

(b) Use the substitution $u = \ln x$ to find the exact value of **3**

$$\int_e^{e^2} \frac{dx}{x \ln x}$$

(c) Evaluate $\lim_{x \to 0} \frac{\sin 3x}{4x}$ **1**

(d) Sketch on the same diagram $y = \frac{1}{x}$ and $y = \sqrt{x}$. **3**

Hence, or otherwise, solve $\frac{1}{x} \ge \sqrt{x}$.

(e) Find $\int \sin x \cos x \; dx$. **2**

QUESTION 3 *Start a new sheet of paper. Write your number on the top of the page.* **MARKS**

(a) Write down the zeros of the polynomial function **3**

$$P(x) = (x-3)(x-1)(x+1)(x+2).$$

Sketch a graph of $P(x)$, showing the x and y intercepts.

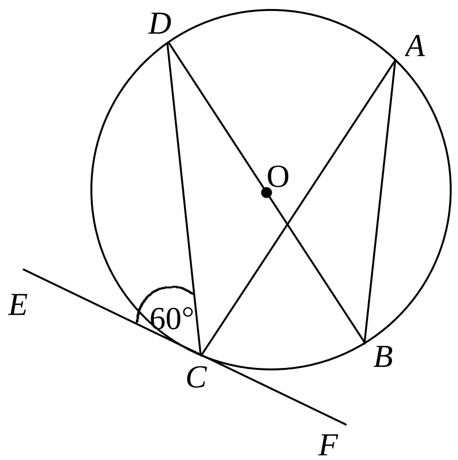
(b) Express $\cos\theta - \sin\theta$ in the form $r\cos(\theta + \alpha)$ where r is a positive number. Hence or otherwise solve the equation $\cos\theta - \sin\theta = 1$. **5**

(c) Find the area under the curve $y \quad \frac{2x}{x^2 +}$ between $x = 0$ and $x = 2\sqrt{2}$. **2**

(d) If $f(x + 2) = x^2 + 2$, find $f(x)$. **2**

QUESTION 4 *Start a new sheet of paper. Write your number on the top of the page.* **MARKS**

(a) A, B, C and D are points on the circumference of a circle with centre O. EF is a tangent to the circle at C and the angle ECD is 60°. **3**



(i) Copy this diagram onto your examination page.

(ii) Find the value of $\angle BAC$ giving reasons.

QUESTION 4 ***(continued)*** **MARKS**

(b) Prove, by mathematical induction, that **4**

$$\frac{1}{1\times 4}+\frac{1}{4\times 7}+\frac{1}{7\times 10}+\cdots+\frac{1}{(3n-2)(3n+1)}=\frac{n}{3n+1}$$

for all positive integer values of n.

(c) Find all solutions of the equation $\sin 2x = \cos x$. **2**

(d) Find the value of the constants p and q if $x^2 - 4x + 3$ is a factor of **3**

$$P(x) = x^3 + px^2 - x + q.$$

QUESTION 5 *Start a new sheet of paper. Write your number on the top of the page.* **MARKS**

(a) Newtons Law of Cooling can be expressed in the form $\frac{dT}{dt} = -k(T-P)$, where P is the temperature of the surrounding medium and t is the time. **6**

(i) Verify by substitution that $T = P + Ae^{-kt}$, where A is a constant, is a solution to the above differential equation.

(ii) A body whose temperature is 170°C is immersed in a liquid kept at a constant temperature of 40°C. In 45 minutes the temperature of the immersed body falls to 105°C. Find the exact temperature of the body in another 90 minutes, and the time when the temperature of the body reaches 80°C (to the nearest minute).

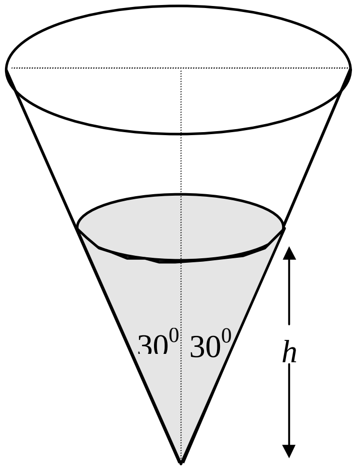
(b) If α and β are the roots of the quadratic equation $x^2 + px + q = 0$, write down (in terms of p and q) the values of: **2**

(i) $\alpha + \beta$

(ii) $\alpha^2 + \beta^2$

QUESTION 5 (*continued*) **MARKS**

(c) A hollow cone with a vertical angle of 60° is held with its axis vertical and vertex downwards. **4**



Water is being poured into the cone at a uniform rate of 12 cubic metres per second.

(i) Show that when the water level has reached a height of h metres, the volume of water in the cone in cubic metres is given by $V = \frac{1}{9}\pi h^3$

(ii) Find the rate at which the water level is rising when the water has reached a height of 6 metres (leave your answer in terms of π)

QUESTION 6 *Start a new sheet of paper. Write your number on the top of the page.* **MARKS**

(a) Prove that $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d^2x}{dt^2}$ where x is the displacement of a particle travelling at a velocity of v at time t. **2**

(b) Show that the motion of a particle whose velocity is given by $v^2 = 4 + 24x - 4x^2$ is simple harmonic. **5**

For such a particle, determine:

(i) the period of the motion

(ii) the amplitude

(iii) the maximum velocity of the particle

QUESTION 6 (***continued***) **MARKS**

(c) (i) Differentiate $y = 3^x$. **3**

(ii) Hence find $\int 3^x \, dx$.

(d) Find $\int \sin^2 3x \, dx$. **2**

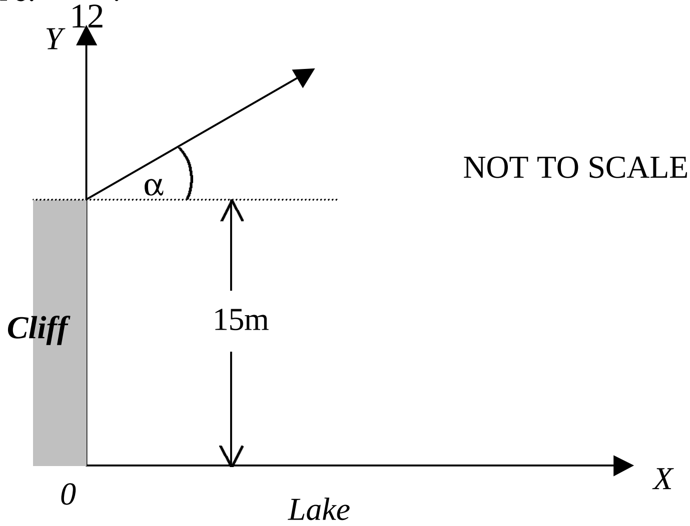
QUESTION 7 *Start a new sheet of paper. Write your number on the top of the page.* **MARKS**

(a) (i) Differentiate xe^{1-x}. **4**

(ii) Hence evaluate $\int_0^1 xe^{1-x} \, dx$.

(b) Find the value of $\sin^4 \alpha - \cos^4 \alpha$ if $\sin^2 \alpha - \cos^2 \alpha = 0$. **2**

(c) A stone is projected with a velocity of 26 metres per second at an angle of α to the horizontal from the top of a vertical cliff 15 metres high, overlooking a lake. It is given that $\tan \alpha = \frac{5}{12}$. **6**



The equations of motion of the stone (with air resistance neglected) are

$$\ddot{x} = 0 \quad \text{and} \quad \ddot{y} = -g \, .$$

(i) By taking the origin, *0*, to be the base of the cliff show that the horizontal and vertical components of the stone's displacement from the origin after t seconds is given by: $x = 24t$ and $y = -\frac{1}{2}gt^2 + 10t + 15$.

<u>QUESTION 7</u> **(*continued*)**

(ii) Hence, or otherwise, calculate the time which elapses before the stone hits the lake and find the horizontal distance of the point of contact from the base of the cliff. (Assume the approximate value of 10 m/s^2 for g).

(iii) What is the maximum height reached by the stone?