THE BINOMIAL THEOREM – WORKSHEET

COURSE/LEVEL

NSW Secondary High School Year 12 HSC Extension Mathematics. Syllabus reference: 17.1 – 17.3

1. Expand $\left(2x^3 + y\right)^4$ using the binomial theorem.

2. (a) For the binomial expansion of $(a+b)^n$ write down the $(k + 1)$th term, T_{k+1}.

   (b) Hence find the coefficient of x^3 in the expansion of $\left(x^2 - \dfrac{2}{x}\right)^8$.

3. (a) Factorise $1 + x + x^2 + x^3$.

   (b) Hence, or otherwise, show that the coefficient of x^4 in the expansion of $\left(1 + x + x^2 + x^3\right)^3$ is 12.

4. Show that the numerically greatest coefficient in the expansion of $(2+3x)^{10}$ is 2 449 440. Also show that the greatest term when $x = -2$ is ${}^{10}\mathrm{C}_8\, 2^2\, 6^8$.

5. Using the expansion of $(1+x)^n$, show that $2^n = \sum_{k=0}^{n}\binom{n}{k}$.

6. Differentiate both sides of the identity

$$(1+x)^{2n} = \sum_{k=0}^{2n}\binom{2n}{k}x^k$$

   and show that

$$\sum_{k=0}^{2n}k\binom{2n}{k} = n\,4^n$$

7. By evaluating $\int_0^2 (1-x)^{2n+1}\,dx$ in two different ways, prove that the following identity holds for all odd values of m:

$$2 - \frac{2^2}{2}\,{}^m\mathrm{C}_1 + \frac{2^3}{3}\,{}^m\mathrm{C}_2 - \ldots + (-1)^r\frac{2^{r+1}}{r+1}\,{}^m\mathrm{C}_r + \ldots + (-1)^m\frac{2^{m+1}}{m+1}\,{}^m\mathrm{C}_m = 0$$

8. Using the expansion $(1+x)^n = \sum_{k=0}^{n}\binom{n}{k}x^k$ prove the following:

   (i) $1 - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \ldots + (-1)^n\binom{n}{n} = 0$

   (ii) $1 - \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} - \ldots + \frac{(-1)^n}{n+1}\binom{n}{n} = \frac{1}{n+1}$.