COMPLEX NUMBERS – WORKSHEET #1

COURSE/LEVEL

NSW Secondary High School Year 12 HSC Mathematics Extension 2.

TOPIC

Complex Numbers: Arithmetic of complex numbers and solving quadratic equations. (Syllabus Ref: 2.1)

1. Express each of the following in the form $a+ib$ where a and b are real.

   (i) $(1+i)^2$ (ii) $\dfrac{1}{1+i}$ (iii) $\dfrac{i}{(1+i)^2}$ (iv) $\left(\dfrac{1+i}{1-i}\right)^2$

2. Given that $z=3+4i$ and $w=3+i$. find the following in the form $x+iy$.

   (i) $z+w$ (ii) $z-w$ (iii) zw (iv) $\overline{w}$ (v) $\dfrac{z}{w}$ (vi) $\dfrac{w}{z}$

3. If $z=1+i\sqrt{3}$, express each of the following complex numbers in the form $a+ib$ where a and b are real numbers.

   (i) $\overline{z}$ (ii) $z\overline{z}$ (iii) z^2 (iv) $\dfrac{1}{z}$

4. Find the values of the real numbers a and b if $\dfrac{a}{1-i}+\dfrac{b}{2+i}=1$.

5. Prove that $\mathrm{Re}(z)=\dfrac{z+\overline{z}}{2}$ and $\mathrm{Im}(z)=\dfrac{z-\overline{z}}{2}$ for all values of z.

6. If two complex numbers are such that their sum and their product are both real, show that they are conjugate.

7. Find the square roots of the following numbers and express your answer in the form $x+iy$.

   (i) -5 (ii) $-5i$ (iii) $\sqrt{i}$ (iv) $\dfrac{1+i}{1-i}$ (v) $1+\dfrac{2}{i}$ (vi) $5+2i$

8. Show that $(1+3i)^2=6i-8$. Hence solve the equation $2z^2-(3+i)z+2=0$.

8. Find all integers x and y such that $(x+iy)^2=-3-4i$. Hence or otherwise solve the equation $z^2-7z+13+i=0$.

10. Solve $2z^2-6iz-3=0$.

11. Solve $4z^2+(3-4i)z-(1-i)=0$.

12. Solve the following equations for z. Express your answers in the form $x+iy$.

    (i) $z^2+z+1=0$ (ii) $\dfrac{2z}{2+i}+3-i=(2-i)z$