COMPLEX NUMBERS – WORKSHEET #4

COURSE/LEVEL

NSW Secondary High School Year 12 HSC Mathematics Extension 2.

TOPIC

Complex Numbers: Curves and regions. (Syllabus Ref: 2.5)

1. Sketch the locus of the point P representing the complex number z, on an Argand diagram, for each of the following:

   (a) $\operatorname{Re}(z)=2$

   (b) $\operatorname{Re}(z)=\operatorname{Im}(z)$

   (c) $|z-1-3i|=2$

   (d) $|z-1|=|z+2-i|$

   (e) $\arg(z-i)=\frac{\pi}{4}$

   (f) $\arg\left(\frac{z-i}{z-2}\right)=\frac{\pi}{3}$

   (g) $|z-2|=\operatorname{Re}(z)$

   (h) $|z-2|+|z+2|=6$

   (i) $|z-2|-|z+2|=2$

   (j) $|z-3|=2|z|$

2. Sketch the locus of z if $|\operatorname{Re}(z)|+|\operatorname{Im}(z)|=1$.

3. If $(z-\bar{z})^2+8(z+\bar{z})=16$ show that the locus of z is a parabola.

   (i) Sketch the parabola

   (ii) Show that $-\frac{\pi}{4}\le \arg z\le \frac{\pi}{4}$

4. Sketch the locus of the points P representing z in each case:

   (a) $1<\operatorname{Re}(z)<2$

   (b) $\left|\frac{z-1}{z+1}\right|\le 1$

   (c) $1\le |z+2+i|\le 2$

   (d) $|z-1|<|z-i|$

   (e) $\operatorname{Im}\left(\frac{1}{z}\right)<-\frac{1}{2}$

   (f) $(1-i)\bar{z}=(1+i)z$

   (g) $|z-1|=|z+2|=|z-i|$

   (h) $|z+2i|=\operatorname{Re}(z)+2i$

   (i) $\frac{\pi}{3}<\arg(z-i)\le\frac{\pi}{2}$

   (j) $\frac{1}{4}<\operatorname{Re}\left(\frac{\bar{1}}{z}\right)+\operatorname{Im}\left(\frac{\bar{1}}{z}\right)<\frac{1}{2}$

5. If z satisfies $|z+3i|^2-|z-3i|^2=12$, prove that the locus is the line $y=1$.

6. If z satisfies $|z+ik|^2+|z-ik|^2=10k^2,\ k>0$, prove that the locus of z is a circle, centre 0, radius $2k$.

7. $P_1,\ P_2,\ P_3$ represent the complex numbers $z_1,\ z_2,\ z_3$ respectively where $z_1z_3=z_2{}^2$. Show geometrically that OP_2 bisects the angle P_1OP_3.

\