GRAPHS – WORKSHEET #4

COURSE/LEVEL

NSW Secondary High School Year 12 HSC Mathematics Extension 2.

TOPIC

Graphs: General approach to curve sketching (Syllabus Ref: 1.8)
Using graphs (Syllabus Ref: 1.9)

1. Sketch the graph of the curve defined by the equation $x^2 + y^2 + 4xy + 3 = 0$, showing features including *x*– and *y*– intercepts and the coordinates of any stationary and critical points.

2. Sketch the graph of $x^2 + y^2 - xy = 3$, showing *x*– and *y*– intercepts, critical points and stationary points.

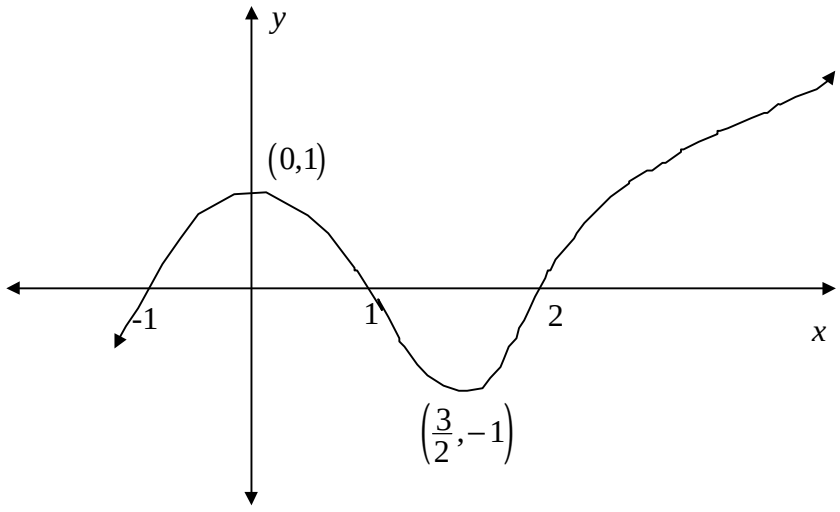
3. Sketch the graph of $x^2 + 9y^2 = 9$, showing *x*– and *y*– intercepts, critical points and stationary points.

4. Sketch the graph of $y = x^x$, showing *x*– and *y*– intercepts, critical points and stationary points. (Hint: use implicit differentiation to find the derivative of ln *y*.)

5. Sketch the graph of $x^2 + y^2 - x^2y^2 = 0$.

6. Sketch the graph of $\sqrt{x-1} + \sqrt{y} = 1$.

7. Use the graph of the function $y = f(x)$ sketched below to write down the range of values of *c* for which the equation $f(x) + c - 2 = 0$ has three distinct roots. The co-ordinates of the two turning points are given.



8. Sketch the graph of $y = x\left(x^2 - 4\right)$. Find the set of values of the real number c if the number of real roots of the equation $x\left(x^2 - 4\right) + c = 0$ equals:

(i) 3 (ii) 2 (iii) 1

9. (i) Sketch the function $\mathrm{f}(x) = x^3 + 1$.

(ii) On the same set of axes, sketch the function $\mathrm{g}(x) = \dfrac{1}{x^3 + 1}$. Clearly indicate on your sketch the equations of the asymptotes and the co-ordinates of any stationary points or points of inflection of $y = \mathrm{g}(x)$.

(iii) Find the values of x for which $x^3 + 1 < \dfrac{1}{x^3 + 1}$.

10. Consider the graph of $y = x^3 - kx^2 + k$, for $k > 0$.

(i) Find the co-ordinates of the stationary points and determine their nature.

(ii) By examining the graph of $y = x^3 - kx^2 + k$ (for varying *positive* values of k), show that $x^3 - kx^2 + k = 0$ has 3 distinct roots if $k > \dfrac{\sqrt{27}}{2}$.

11. Sketch the graph of $y = \dfrac{(x-1)^3}{x^3 - 1}$. Hence, or otherwise, determine the range of values of the real number c if the equation $(x-1)^3 = c\left(x^3 - 1\right)$ has no real roots.

12. Sketch the graph of $y = x(x-1)(x-2)$. Find the set of values of the real number k if the number of real roots of the equation $x(x-1)(x-2) - kx = 0$ equals:

(i) 2 (ii) 3