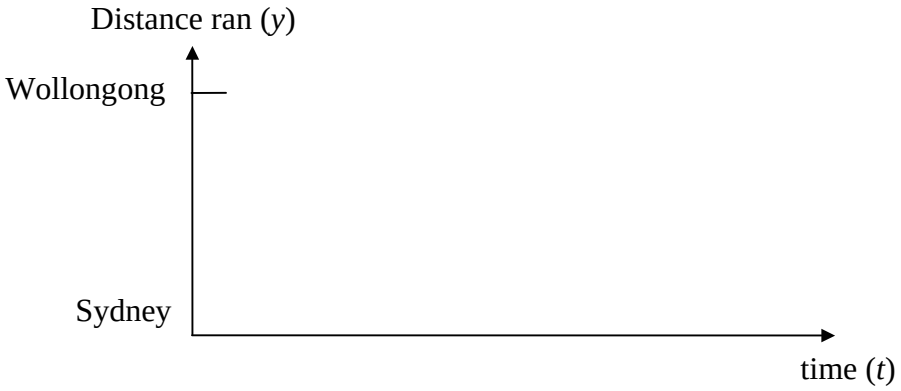
INTERPRETING THE SECOND DERIVATIVE – WORKSHEET

COURSE/LEVEL

NSW Secondary High School Year 12 HSC Mathematics.
Syllabus reference: 10.1, 10.4.

Illustrate each of the following vignettes by drawing an appropriate sketch. Let t be the independent variable, and y its dependent variable so that the graph you sketch has equation $y = \mathrm{f}(t)$.

1. In a race from Sydney to Wollongong, the race leader runs without stopping. The closer she gets to Wollongong the faster she runs. Draw a graph to show over time the distance she covers as the race progresses.



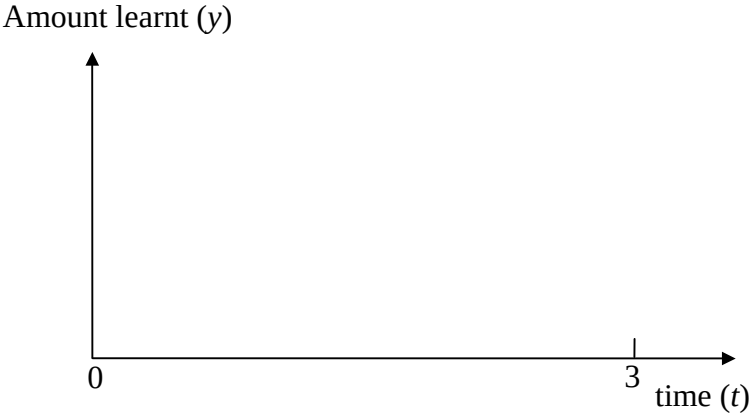
2. A tree continually increases in height over the summer. As the summer's end draws near, the tree slows its growth. Draw a graph showing the height of the tree as a function of time.

3. A skydiver jumps from an airplane at 10,000 feet. The parachute fails to open and the skydiver falls to the ground at an increasing speed. Sketch a graph of the skydiver's altitude as a function of time.

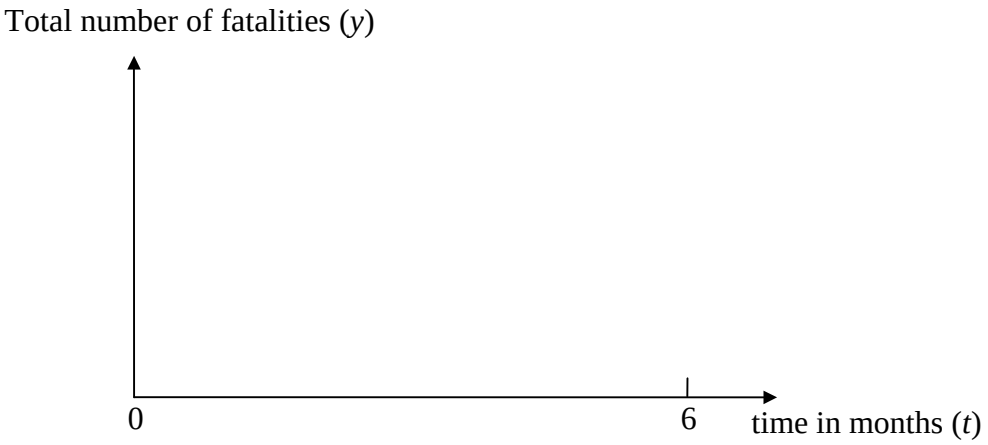
4. A lady of advanced age climbs a steep hill. The further she progresses up the hill, the slower she climbs. Draw a graph to show the relationship between the distance climbed and the time taken.

5. After taking a bath, you pull the plug and allow the water to disappear down the drain. Sketch a graph to show the changing water level.

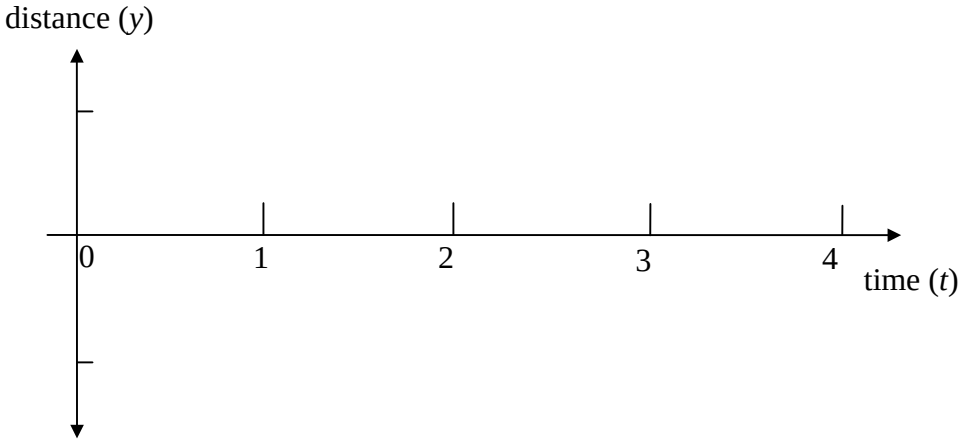
6. A student spends three hours studying for a test. She finds, during this time, that as she studies, she accumulates an increasing amount of knowledge. At the end of this period of study, she observes that she learnt more in the first hour than in the second, and more in the second than in the third. Draw a sketch showing how much she learnt.



7. An island population is subject to a fatal virus. For the first three months the fatality rate increases until a vaccine is discovered which reduces the fatality rate so that, by the end of six months, no fatalities are recorded. Draw a graph to show the total number of fatalities at any point of time up until the end of six months.



8. Imagine a long and heavy pendulum swinging to and fro. Initially the pendulum is at the centre of its motion. It takes one second before it turns around and another second to return to the centre. Draw a sketch showing the displacement of the particle from the centre of its motion in the first 4 seconds.



9. Sketch the gradient function for each of your graphs in Questions 1 – 7 (using the above axes).

10. For each of the graphs in Questions 1 - 8, label the sections where $f''(t) > 0$, $f''(t) < 0$ or $f''(t) = 0$.