I. M.R. Pinheiro[1]

NONISOTHERMAL NEWTONIAN FILM BLOWING - INEQUALITIES DEVELOPMENT

ABSTRACT. In this paper, we present some scope of solutions to a simplified equation of the system previously presented by us in [14], equation obtained when disregarding gravity.

2000 Mathematical Subject Classification. 34E10.
Key words and phrases. nonisothermal, film, blowing, newtonian,inequality..

1. Problem formulation

Temperature changes seem to influence how accurate the model, describing the process of film blowing, from die to die, is. Having into sight what is mentioned in [11], we can see what major improvements are obtained by considering temperature changes that got disregarded by the isothermal models such as Tam's model [2]. There are a few temperature equations available in the present scientific community material, such as Han and Park's, Alaie's, etc.
In [11], the model under analysis is for viscoelastic fluids, is described as being the 'Kelvin model', and it makes use of a reasonably different temperature equation when compared to ours. However, what is interesting for us there is the experimental data, which we use here.
We have made an analysis of Tam's results ([2]) and found out that it is actually hard to get his equations working without using a negative initial slope for the radius curve, what makes the model inadequate, or not as adequate as a model which does not hold such a problem. However, we may allow room for other possible values, not detected by us, being

[1]mrpprofessional@yahoo.com

able to generate a faithful model. Via analyzing what may be implied taking the model as accurate, we may reach a definite proof, analytical, of its inadequacy or, otherwise, more evidence in its favor.

Tam's results should also derive from Han and Park's work, just like ours. However, because our equations are easier to manipulate, include the temperature term, and give faithful bubble profiles for positive initial slopes of the radius curve, we have decided to drop Tam's equations in favor of ours.

The intention is obviously reaching a stage when mathematical results might become interesting for the industry in a way that it is worthwhile investing on them. If we can reach a close-to-100% description of the actual results in the process, our results might turn out to be of interest to the industry who, until nowadays, thinks that the waste of the material in the process is negligible. The waste under analysis, in this case, is obviously the waste of polymer melt, once bad bubble formation leads to failure of the process and waste of material [2].

What appears in [14] is a comparison between the isothermal newtonian model and the nonisothermal newtonian model, so that the reader acquires 'choice power' (is more equipped to decide which model is most adequate in terms of describing each of the variables of interest in the problem: radius, velocity, and temperature).

Our conclusion was that the nonisothermal model is better for the radius profile but is worse for the velocity profile and, trivially, better for the temperature profile.

Having this into sight, we develop some analytical incursions on one of the simplified equations, which appears as a result of our nonisothermal system after ignoring gravity, as previously done by Tam in [2]. We

[2]We have made some inquiries to plastic manufacturers who all gave us this information.

achieve inequalities which provide some constraints for solution trials, that is, we define a region for the perturbational work (see, for instance, Shepherd[3] et al, for example of pertubational work).

2. State of the art

Han and Park ([1]) obtained some results on nonisothermal newtonian fluids regarding them as a restriction of the power-law fluids. Tam ([2]) took a completely newtonian approach, his results dealing with the isothermal situation. Alaie and Papanastasiou ([3]) considered the nonisothermal situation of film blowing and treated it with an integral constitutive model. Kanai and White ([4]) produced an experimental study on the stability of nonisothermal (temperature dependent viscosity) film blowing of viscoelastic newtonian melts. Yamane and White ([5]) researched on the significance of non-newtonian viscosity on non-isothermal film blowing. From the empirical observation, one may get dynamic viscosity written as a function of temperature and frequency, wall shear stresses, pressure losses, wall-slip coefficients, shear and temperature dependent viscosities ([5]).

We now present these results from a different point of view, however: we use a different scaling, which is supposed to facilitate calculations, and prove that the system of equations, expressing the process of nonisothermal newtonian film blowing, should be split into three other systems. As a side result, we present an alternative to the Force-balance equation, as obtained by Han and Park. In this work, we depart from a generic situation of film blowing, providing all generic tensors and parameters, to then work with power-law film blowing (manufacture of plastic from

[3]John Shepherd, RMIT, AU, 2003, see conference papers up to the date, for instance.

fluids that obey the power-law rule), which then gets refined into newtonian film blowing. In short, this study provides a model for nonisothermal newtonian film blowing.

For the next table, we base ourselves in [11], just changing one line of [11]'s table, the line regarding Han and Park's work: That is because we consider their newtonian proposal, included in the power-law proposal, instead of the power-law one, which disregards their model. We strongly advise the reader to consult the sources mentioned in [11], which we paste here with some minor corrections, for righter history regarding research in the field. The present state of the art is:

Author/s	Model description	Limitations
Pearson and Petrie [12, 13]	Isothermal Newtonian	Did not incorporate the non-Newtonian flow behavior of polymer melts
Han and Park [7]	Non-isothermal newtonian	Did not account for viscoelasticity
Kanai and White [4]	Non-isothermal Newtonian with crystallization	Did not allow for non-Newtonian behavior of fluid
Sidropoulos et al. [11]	Modified non-isothermal Newtonian	Did not allow for viscoelastic nature of polymer melt
Petrie [11]	Non-isothermal Newtonian and isothermal purely elastic model	Did not allow for the viscoelastic response of materials

Table 1. 1

3. Some analytical incursions in the solution of the bubble system

We here deal with the simplified equation obtained in [15] by means of inequalities, call it Eq. 1.

$$2C^2r''r^2(f_o-B(r^2-1))+r(f_o-B(3r^2-1))(1+C^2(r')^2)-5Cr'e^{\beta(\frac{1}{s(z)}-1)}=0$$

PRIMARY DEVELOPMENTS

We take our suggested approach to the problem and consider that the minimum the function $r(z)$ reaches is $r(0)=1$ and the maximum is $r(1)=A$. The radius will always be a positive measure. Therefore:

$$0\leq r(0)\leq r(z)\leq r(1),$$

or

$$1\leq r\leq A.$$

On top of the above information, we know that the bubble problem belongs to the scope of the Applied Problems in Mathematics, that is, it is a severely forced fit between the perfect world of Mathematics and the so inaccessible transcendent reality (transcending any possible human description, part of the own God, for the perfect description of nature elements will never be accessible by human beings, for it is impossible to communicate all the precise complexity of anything in nature to another human being. Like God, nature is something which transcends human communication and it is only accessible via direct connection, depending on the ability of the individual to connect with it (see [17] for instance)). All this prologue was to introduce the idea of accepting, and incorporating, experimental values into the mathematical calculations as if they had been attained via abstract mathematical deduction. Once it is impossible to determine those values via abstraction without escaping reality completely, it is either the case that we never apply Mathematics to it, or we accept holding less deterministic approaches in order to get at least part of the work, maximum possible, performed via abstract mathematical tools. The mixed approach is well-accepted by many mathematicians, and it ended up included in major theories relating differential equations, for instance. It is also common for Abstract Mathematics, for instance, that we 'guess' the roots of a hard-to-work-with equation, and we then just confirm that to be the case, via algebraic calculations. In our problem, we have then made use of the computer to 'guess' the values for the constants involved, what may be regarded as equivalent to the guessing process for roots of equations. From that guess, we have plotted the graph, and compared with the reality of the bubble for the Industrial Process, therefore establishing the connection with the actual life model, which is the major intention. With that done, we should then bring the results from [15] to the present work. There, we found out that A, B, and C should all be close to zero and less than one, all non-negative. With those results, we shall proceed to a few inferences, which should, then, bear proof in the computer model, which should be, to the best of the human eyes observation, the most accurate picture of the real model as possible. Our programs did not work with 3D-simulations, however, and our simulations were based on those claiming they did perform real life ones (therefore relying on their experimental work).

3.1. **Second derivative.** From Calculus, we recall that wherever a function increases, its first derivative is positive. Wherever a function decreases, its first derivative is negative, or the tangent to the graph of the function in that location. Wherever it is not increasing, or decreasing, the first derivative is zero.

The second derivative measures the variation of the first. Therefore,

wherever the first derivative increases, the second is positive. Wherever the first derivative decreases, the second is negative. Increase, as for first derivative, means going from negative, or null, to positive, or lower slope to higher slope, and these are the situations in which the second derivative will be non-negative.
Notice that our function suffers from two stages only, as for its profile we consider: stable in a value, or increasing towards the vertical Z axis. Therefore, the first derivative is either zero or positive. The second derivative, however, will observe the behavior of the first. Whilst the bubble is doing the first turn, it is also suffering of increase in the tangent value, that is, the second derivative is positive. When the bubble profile is expanding, however, the first derivative is decreasing, up to zero, that is, the second derivative is going negative to zero.
We have named those little pieces of the curve via naming a the first turn and c the second. This way:

- EVEN: $r'' = r' = 0$ for $z \in [0, a]$, $z \in [d, 1]$;
- G.H.: $r' \geq 0$, $r'' \geq 0$ for $z \in (c, d]$;
- G.L.: $r' \geq 0$, $r'' \leq 0$ for $z \in (a, c)$.

For EVEN:
$r(f_o - B(3r^2 - 1)) = 0 \implies$ Once $r \neq 0$, we have: $f_0 = B(3r^2 - 1)$

$$\therefore 2B \leq f_o \leq B(3A^2 - 1),$$

as only possible inference from equation. $r(z) = 1$, or $r(z) = A$, in those intervals. The contribution of the finding is then tying the solution further.
For both (G.H.) and (G.L.) situations, we may assign:

- $$T_1 = 2C^2 r'' r^2 (f_o - B(r^2 - 1));$$

- $$T_2 = r(f_o - B(3r^2 - 1))(1 + C^2(r')^2);$$

- $$T_3 = -5Cr'e^{\beta(\frac{1}{s(z)}-1)}.$$

Remark 1. Interesting enough to notice that half of the profile of the bubble is convex, but the other half is concave. If we prefer, there might be an s fitting the whole lot so that the bubble is $S-$convex up to a certain value of A (and it is obviously less likely that we add mistake to the analysis if we take the whole lot instead of a single piece of it).

Basically, we then have to split the problem into two parts: one where the second derivative is non-negative (G. H.), another where the second derivative is non-positive (G. L.), with the first derivative always non-negative (once the tangent remains in the first quadrant).

(1) (G.H.):
   In this case, once C is non-negative, but less than one (experimental), B is non-negative and less than one as well, we have:
   - $0 \leq 2C^2r''r^2 \leq 2r''A^2$. As for $r^2 - 1$, we have: $0 \leq r^2 - 1 \leq A^2 - 1 \implies f_o \geq f_o - B(r^2 - 1) \geq f_o - B(A^2 - 1)$, or $f_o - B(A^2 - 1) \leq f_o - B(r^2 - 1) \leq f_o$. This implies: $0 \leq 2C^2r''r^2(f_o - B(r^2 - 1)) \leq 2r''A^2f_o$. As a result: $0 \leq T_1 \leq 2r''A^2f_o$;
   - $1 \leq (1 + C^2(r')^2) \leq 1 + (r')^2$. Also: $2B \leq B(3r^2 - 1) \leq B(3A^2 - 1) \implies -2B \geq -B(3r^2 - 1) \geq -B(3A^2 - 1)$. As a result: $A(f_o - 2B) \geq r(f_o - B(3r^2 - 1)) \geq (f_o - B(3A^2 - 1))$, or $(f_o - B(3A^2 - 1)) \leq r(f_o - B(3r^2 - 1)) \leq A(f_o - 2B)$. Finally: $(f_o - B(3A^2 - 1)) \leq r(f_o - B(3r^2 - 1))(1 + C^2(r')^2) \leq$

$A(f_o - 2B)(1 + (r')^2)$, or $(f_o - B(3A^2 - 1)) \leq T_2 \leq (f_o - 2B)(1 + (r')^2)A$.

- The temperature starts hot and goes to 'frozen' state. With our scaling and etc., we got $s(0) = 1$ as a starting value for our temperature. Therefore, it is true that: $0 \leq \frac{1}{s(z)} - 1 \implies 0 \leq \beta(\frac{1}{s(z)} - 1) \implies 1 \leq e^{\beta(\frac{1}{s(z)}-1)} \implies -5Cr' \geq -5Cr'e^{\beta(\frac{1}{s(z)}-1)}$, that is:
  $-5Cr' \geq T_3$. Notice that $s(z)$ will vary from 1 to 0. This way, it is becoming little and, with that, it is making the ratio go to infinity, what makes the whole expression go to negative infinity, what does not provide us with a bound for below in this case;
- As a final result for our G. H., we find:
  (In. X)
  $T_1 + T_2 + T_3 \leq 2r''A^2 f_o + (f_o - 2B)(1 + (r')^2)A - 5Cr'$.

(2) (G.L.):

In this case,

- $2r''A^2 \leq 2C^2r''r^2 \leq 0$. As for $r^2 - 1$, we have: $0 \leq r^2 - 1 \leq A^2 - 1 \implies f_o \geq f_o - B(r^2 - 1) \geq f_o - B(A^2 - 1)$, or $f_o - B(A^2 - 1) \leq f_o - B(r^2 - 1) \leq f_o$. This implies: $2r''A^2(f_o - B(A^2 - 1)) \leq 2C^2r''r^2(f_o - B(r^2 - 1)) \leq 0$. As a result: $2r''A^2(f_o - B(A^2 - 1)) \leq T_1 \leq 0$;
- $1 \leq (1 + C^2(r')^2) \leq 1 + (r')^2$. Also: $2B \leq B(3r^2 - 1) \leq B(3A^2 - 1) \implies -2B \geq -B(3r^2 - 1) \geq -B(3A^2 - 1)$. As a result: $f_o - 2B \geq (f_o - B(3r^2 - 1)) \geq (f_o - B(3A^2 - 1))$, or $r(f_o - B(3A^2 - 1)) \leq r(f_o - B(3r^2 - 1)) \leq r(f_o - 2B)$. Finally: $r(f_o - B(3A^2 - 1)) \leq r(f_o - B(3r^2 - 1))(1 + C^2(r')^2) \leq r(f_o - 2B)(1 + (r')^2)$, or $r(f_o - B(3A^2 - 1)) \leq T_2 \leq r(f_o - 2B)(1 + (r')^2)$.

- From the previous case, G. H., to this one, nothing changes regarding the temperature. Therefore, our conclusion is: $-5Cr' \geq T_3$, as before;
- As a final result for our G. L., we find:
  (In. Y)
  $T_1+T_2+T_3 \leq r(f_o-2B)(1+(r')^2)-5Cr' \leq r(f_o-2B)(1+(r')^2)$.

Summarizing and joining:

- (In. 0)
  EVEN: $r''=r'=0$ for $z \in [0,a]$, $z \in [d,1]$:

$$2B \leq f_o \leq B(3A^2-1),$$

  with $r(z)=1$ for $z \in [0,a]$, and $r(z)=A$, for $z \in [d,1]$;
- (In. X)
  G.H.: $r' \geq 0$, $r'' \geq 0$ for $z \in (a,c)$:
  $T_1+T_2+T_3 \leq 2r''A^2f_o+(f_o-2B)(1+(r')^2)A$;
- (In. Y)
  G.L.: $r' \geq 0$, $r'' \leq 0$ for $z \in (c,d]$;
  $T_1+T_2+T_3 \leq r(f_o-2B)(1+(r')^2)$, and if $f_o \leq 2B$, then $T_1+T_2+T_3 \leq (1+(r')^2)A$, but if $f_o \geq 2B$, then $T_1+T_2+T_3 \leq (f_o-2B)(1+(r')^2)A$.

With all the above reasoning, we may establish some reasonable new analytical bounds and determine them, in terms of the solution for the equation. With a few perturbational techniques, we reach a reasonable bound for the solution. We then suggest further perturbational work to reach the exact solution for the equation.

4. Solution to the equations that are bounds

- For (In. X), we get:
  $2r''A^2 f_o + (f_o - 2B)(1 + (r')^2)A = 0 \implies r'' = -\frac{(f_o-2B)(1+(r')^2)A}{2A^2 f_o}$, that is, $r'' = -\frac{f_o-2B}{2Af_o} - \frac{(f_o-2B)(r')^2}{2Af_o}$, which we are going to solve later on in this very paper, and we will refer to it as Eq. X;
- For (In. Y), we get[4]: $(f_o - 2B)(1 + (r')^2)A = 0 \implies (r')^2 = \frac{2B-f_o}{f_o-2B} = -1 \implies r' = \pm i$, what is not acceptable. However, we can majorize the upper bound in order to achieve a real result. If $(f_o - 2B) < 0$, that is, $f_o < 2B$, we do: $(f_o - 2B)(1 + (r')^2) - (f_o - 2B) = 0 \implies (r')^2 = 0$ or we assert that the constants are tied. This will imply $r' = 0 \implies r(z) = k$. If ever working with definite integral and the interval, we actually get: $r(1) - r(0) = k \implies A - 1 = k \implies A = k_*$, what is no real progress, but consistent. And if $f_o > 2B$, we add r' to the expression to majorize it, getting $(f_o - 2B)(1 + (r')^2)A = -r' \implies r' = (2B - f_o)(1 + (r')^2)A$, which we shall solve later on in this paper, and we shall call this last equation, Eq. Y.

Our progresses then, with our bounds, seem to point at analytical approximations, which lead to better understanding of the constants involved, if nothing else. We are also left with solving the equations for the bounds:

- (Eq. X)
$$r'' = -\frac{f_o - 2B}{2Af_o} - \frac{(f_o - 2B)(r')^2}{2Af_o};$$
- (Eq. Y)
$$r' = (2B - f_o)(1 + (r')^2)A.$$

We notice the most important factor, for our solutions, regarding the bounds, is $|2B - f_o|$.

[4]Iff $f_o \neq 2B$.

We then hold two possible situations for the inequalities above (for if $f_o = 2B$, we are in EVEN): either $f_o - 2B > 0$ (and, therefore, $f_o > 2B$), or $f_o - 2B < 0$ (and, therefore, $f_o < 2B$). Call the former, Case 1, and, the latter, Case 2.

Assuming either Case 1 or Case 2, leads us to:

- (**Eq.** X)

$$r'' = -\frac{f_o - 2B}{2Af_o} - \frac{(f_o - 2B)(r')^2}{2Af_o}.$$

$$2r''Af_o + f_o - 2B + (f_o - 2B)(r')^2 = 0.$$

Now, we do $v = r'$, for calculation purposes, to get:

$$(Eq.O) \quad 2v'Af_o + (f_o - 2B)(v^2 + 1) = 0.$$

$$2Af_o dv + (f_o - 2B)(v^2 + 1)dz = 0 \iff \int \frac{dv}{1 + v^2} = \frac{2B - f_o}{2Af_o} z + k$$

$$\therefore \arctan v = \frac{2B - f_o}{2Af_o} z + k \iff \tan\left(\frac{2B - f_o}{2Af_o} z + k\right) = v$$

$$\therefore r(z) = -ln\left|\cos\left(\frac{2B - f_o}{2Af_o} z + k\right)\right| \frac{2Af_o}{2B - f_o} + k.$$

For verification purposes, suffices remembering that $\int \frac{dx}{1+x^2} = \arctan x + k$.

- (**Eq.** Y)

$$r' = (2B - f_o)(1 + (r')^2)A.$$

Same trick as with Eq. X will apply here:

$$v = (2B - f_o)(1 + v^2)A \iff (2B - f_o)Av^2 - v + A(2B - f_o) = 0$$

$$\therefore v = \frac{1 \pm \sqrt{1 - 4(2B - f_o)^2 A^2}}{2A(2B - f_o)}$$

$$r(z) = \frac{1 \pm \sqrt{1 - 4(2B - f_o)^2 A^2}}{2A(2B - f_o)} z + k$$

5. Conclusion

We then have that our radius function will have, according to our calculations, one in three possible behaviors:

- (EVEN) $r'' = r' = 0$ for $z \in [0, a]$, $z \in [d, 1]$: $r(z) = k$ and $2B \leq f_o \leq B(3A^2 - 1)$, with $k \in \{1, A\}$;
- (G. H.) $r' \geq 0$, $r'' \geq 0$ for $z \in (c, d]$:
$$r(z) = -ln\left|\cos\left(\frac{2B - f_o}{2Af_o}z + k\right)\right|\frac{2Af_o}{2B - f_o} + k,$$
what is possible because of the coefficient attached to the logarithm expression, which may be negative;
- (G.L.) $r' \geq 0$, $r'' \leq 0$ for $z \in (a, c)$:
$r(z) \leq \frac{1 \pm \sqrt{1 - 4(2B - f_o)^2 A^2}}{2A(2B - f_o)}z + k$.

Remark 2. Remember here that we have created an artificial bound in order to get a real value (on the issue of second derivative).

References

[1] Han, C.D.; Park, J.Y. Studies on Blown Film Extrusion II. Analysis of the Deformation and Heat Transfer Processes. *Journal of Applied Polymer Science*, 1975, v.19, 3277-3290.

[2] Tam, D.C.H. Mathematical Analysis of the Blown Newtonian Film. Department of Mathematics and Statistics, RMIT, 2003.

[3] Alaie, S.M.; Papanastasiou, T.C. Modeling of non-isothermal film blowing with integral constitutive equations. *Hanser Publishers*, 1993.

[4] Kanai, T. and White, J.L., Dynamics and Stability of the Tubular Film Extrusion of Various Polyethylenes *Polym. Eng. Sci.*, 24 (1984) pp.1185-1201

[5] Yamane, H.; White, J.L. unknown. *Int. Polym. Process.* 1987, 2, p.107.

[6] Pearson, J.R.A. Mechanics of Polymer Processing. *Elsevier Applied Science*, USA, 1985.

[7] Han, C.D.; Park, J.Y. Studies on Blown Film Extrusion I. Experimental determination of Elongational Viscosity. *Journal of Applied Polymer Science*, 1975, v.19, 3257-3276.

[8] Colias, D.I.; Baird, D.G. Polymer processing. *Willey Inter-Science*, USA, 1998.

[9] Tanner, R.I.; Luo, X-L. A computer Study of Film Blowing. *Polymer Engineering and Science*. 1985, v. 25, n.10.

[10] Sheppherd, J.J.; Connell, H.J.;Tam, D.C.H. *An Interior Layer in a Film-Blowing Problem*. 2005, preprint, RMIT.

[11] S.Muke, H. Connell, S.N. Bhattacharya, Numerical modelling and experimental verification of blown film processing, *Journal of Non-Newtonian Fluid Mechanics*, AU, 2003.

[12] J.R.Pearson, C.J.S. Petrie, The flow of a tubular film. Part 2. Interpretation of the model and discussion of solutions, *J. Fluid Mech.*. 1970, v.42, n. 60.

[13] J.R.A. Pearson, C.J.S. Petrie, The flow of a tubular film. Part 1. Formal mathematical representation, *J.Fluid Mech.*. 1970, v.40, n.1.

[14] M.R.Pinheiro, Nonisothermal newtonian film blowing, MMA, August, 2005.

[15] M.R.Pinheiro, Bubble radius function - non isothermal film blowing, *WJMS*. 2007, v. 3, n. 2, pp. 121-129.

[16] P.G. Drazin, Nonlinear systems,*Cambridge University Press*, 1992.

[17] M. R. Pinheiro, RAELISM IS INCONSISTENT AND, THEREFORE, IT SHOULD BE AVOIDED, REJECTED, NOT TO INJURE CROWDS OF POSSIBLE BELIEVERS, online preprint, see $http:$ $www.scribd.com$, under illmrpinheiro.

[18] H. Gauchman, Some Taylor Inequalities Involving Taylor's Remainder. II, *Journal of Inequalities in Pure and Applied Mathematics*, 2003, v. 4, I. 1, article 1.

[19] M. Hazewinkel (Editor), Encyclopaedia of Mathematics, Springer-Verlag Berlin Heidelberg New York, 2002.