Is The Convex Hull About Strawberries?

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Abstract

In this paper, we explain and fix the notion of Polynomially Convex Hull and Hull. We also present the axioms of definition for Calyx and Holder.
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1 Introduction

The polynomially convex hull seems to be a very interesting notion and there are not many people who would not be curious about its plotting, at least. Hull could be seen as the case of some fruits, or seeds, such as the grain of rice, according to [6]. It could also be regarded as the green part in the strawberry, for instance, found described in the literature as 'persistent enlarged calyx at base' (see [5], for instance). Pushing it a bit, the support of the leaves of a flower, the contact of the stem with the leaf. In this sense, one would expect the external case of a set, as in a notebook case, to be present in a 'hull', or the top support of something, once strawberries hang down from the hull, not up, as it may be seen at [17], and at most that, in terms of graphs (the fact that the two options are

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so different should already, for any primary goer in Mathematics, exclude the English word from the language of Science, for Science has got, as most fundamental objective, making people from everywhere on Earth see the same image, as quick as their creator, when reading their material, speed in assessing written theory being one of the most primary objectives of the scientific lingo, ethical duty of it). However, in our preferred notion of it (very different notions were found in the literature researched), the idea is not precisely this one. In the lines that follow, we shall go through a few of the existing references to 'a possible mathematical entity called 'hull'', hoping it is a full stop in the Mathematics for what regards association of the sigmatoid with any branch of it.
The paper is organized as follows:

1. The so many definitions of hull: presentation and analysis;
2. Proposal of axiom of definition for 'calyx' and 'holder' in Mathematics (addition to the mathematical lingo);
3. Fixing of old definitions, those which bear any ethical sense in Mathematics, to what should be, according to well-posedness theory for Philosophy;
4. Conclusion;
5. References.

2 The so many definitions for hull:

Presentation of Definition 1, 'hull':

As defined in [7], for instance, what is meant by polynomially convex hull of K, when K is a compact subset of the complex plane, is:

$$\hat{K} = \{z \in \mathcal{C} : |p(z)| \leq \max_{\zeta \in K} |p(\zeta)| \quad \text{for all polynomials } p\}.$$

A remark, from same source, is: A compact set K is said to be polynomially convex if $K = \hat{\mathrm{K}}$.

Analysis of Definition 1, 'hull':

- Background theory needed:

– What is a polynomial? - this should be the fundamental question for us to understand what is going on with the first definition presented here.

  In [2], we find the following definitions:

  * Definition (Polynomial Definition) Polynomial Expression. A polynomial in x is an algebraic expression that may[1] be written in the form

    $$a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0,$$

    where n is a nonnegative integer, x is a variable, and each of the numbers $a_0, a_1, ...a_n$ are called the coefficients of the polynomials.If $a_n \neq 0$, we say this is a polynomial expression of degree n, a_n is known as the leading coefficient, and a_0 is called the constant coefficient.

  * Definition (Polynomial Definition) Polynomial Function. Any function f that has the form

    $$f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0,$$

    where n is a nonnegative integer and $a_0, a_1, ...a_n$ are real numbers is called polynomial function. If $a_n \neq 0$, we say f is a polynomial function of degree n, a_n is known as the leading coefficient, and a_0 is called the constant coefficient.

- Geometric shape intended: We go with the definition. First we pick a point located in the complex plane, any point. We now apply a polynomial to it. If the polynomial is in the complex numbers set, then the result will be another complex plane point. Now we take the modulus of that result, what means the size of the complex number attained via polynomial, for calculations as if it all belonged to the real numbers, instead. We do that for every member of the compact K, and we get the maximum of that compact after entering, member by member, the polynomial process of transformation, that

[1] We have replaced 'can', from the original extract, with 'may', when reproducing the extract.

is, the function. The maximum may be attained because, in the complex numbers set, theory, it is also true that compacts are sent to compacts (see [16]), if what takes them is a continuous function, and polynomials are continuous functions. Therefore, there will be a maximum, precisely the limit of the compact, so that there are no problems with this specific component of the definition. We consider the modulus of the square of z as our best example, found mentioned in p. 57 of [18]. Notice that, in that case, it is very clear that taking all complex numbers fulfilling the requirement, and calculating their pair via polynomial function, leads us to a shape similar (pushing it quite a lot?) to an inverted strawberry hull for the part above the horizontal plane, with the set called 'hull' keeping on going after that plane, going down the same size it went up from the horizontal plane, trivially.

Another possible interpretation would be that the set K, after going through the operation of 'hull formation', would return a convex complex slice.

- Criticisms:
  - Obvious primary question, derived from the geometric interpretation, is what in a hell does that weird shape have to do with the name 'hull'. As our remarks, in the lines above, clearly point out, there is no way the graphical concept would lead to thinking of a strawberry top. Therefore, the name is simply inadequate, even without considering it is not a word from Mathematics lingo at all, or could, possibly, be allowed to be, for it does have more than one reference in the actual World;
  - Is the polynomially convex hull attached to a particular polynomial function, or a particular polynomial expression? That is a question emerging from simply reading Definition 1. And, independently of whether the previous question was answered with a 'yes' or a 'no', does any of this make any difference?

    From a practical point of view, it does not, for when we find bounds for a function, we look for its image, which is pretty much the polynomial expression in our case so that, in those regards, at least, the definition does not fail. However, it does make a lot of difference from a theoretical point of view, for the well-posedness theory from Philosophy would demand excellence in the matter of ‘posedness’, that is, in the creation, or definition, axioms of the mathematical entities. Maximum specification is implied by the premise of targeting universal understanding. One can see that even with a set of 20 pages of English text to universally define a single word, or problem (see our Sorites discussion, split into two papers of about 20 pages each), there is still room for misunderstanding. Each and every time, therefore, that we can ‘refine’ our mathematical expression, we have ethical duty of doing so, in favor of Science and progress. Therefore, first fixing must be that of completely specifying what we wish to refer to, if function, or polynomial;

– Now, is the hull definition specific to a single polynomial, or not, that is, do we choose the polynomial first and then find its hull, this polynomial being fully determined in terms of its constants, or it is in some other way that we work out the hull? This is also not a trivial question, for the first definition brings the wording ‘for all polynomials p’. It occurs that ‘for all’ is not the same as ‘for each chosen’ and, therefore, may imply that the ‘hull’ is supposed to include all sets of that type for all possible polynomials p at once, what actually trivializes the definition. Therefore, we also need to fix that wording to ‘for each chosen polynomial p’ instead, in order to make the definition something mathematically meaningful;

– If it is the shape we are interested at, of course we should be considering not the complex number, but its image via polynomial function, for the calculation will generate any sort of graph if referring to the complex number instead. As trivial examples, see p. 57 from [18], and draw similar graph for the constant complex function, $f(z) = ci$, $c \in \Re_{+}$. One case will return a convex complex set, but the other will return the whole complex plane with a rectangular hole and, therefore, a non-convex set, or concave. This way, if there were any hope for the word 'convex' to suit this trial of definition, it would have to do with the function graph, not the set of previous domain points;

- The expression 'polynomially convex' should at least refer to two entities in Mathematics, convexity and polynomials. However, even though the definition does bring polynomials in it, there is nothing of convexity in either the hull or the graph of the polynomial, once in our example it does coincide (example mentioned at the previous item on geometry), but suffices considering a third degree polynomial shape and it will not. Of course, this is regarding the definition of convexity trivially extended to more dimensions, what can only be the case, especially in geometric terms...So, there is no sense for the word convexity, or the word hull, in the above mentioned concept, that is, Definition 1. Another point is that any function, defined in the complex environment, compact complex domain, will include the real functions, trivially. Suffices, then, picking simple examples in the reals, say $f(x) = 5$, defined in $[0, 2]$. Our 'intended hull' would then be $(-\infty, 0) \cup (2, \infty)$. Unfortunately, clearly non-convex. $F(x) = 5$ is convex, however. But, then, pick one which is not. Now, proved, in top simplicity, that there is no chance at all for the term 'convex' to mean anything (definition) there which is not solely confusion, mistake...

Final Diagnosis for Definition 1, 'hull':

If we take away the words 'convex', 'hull', and 'for all', and replace 'polynomial' with 'polynomial function', from/in the trial of creation of the entity 'polynomially convex hull', we may be able to fix the axiom of definition to a mathematical palatable level.

The following definitions for hull are also found in the literature ([8]):

Presentation of Definition 2, 'hull':

Definition 1. *For every $E \subset V$, the intersection $ch(E)$, of all convex sets containing E, is a convex set, called the convex hull of E, that is,*

$$ch(E) = \{\sum_0^N \lambda_j x_j; \lambda_j \geq 0; \sum_0^N \lambda_j = 1; x_j \in E; N = 1, 2, ...\};$$

Remark 1. One will also find the notation 'co E' for convex hull.

Analysis of Definition 2, 'hull':

- Background theory needed: The intersection of sets which are convex is not necessarily convex? For instance, if the intersection returns the empty set, is the empty set a convex set? To answer this question, one has to go back to the definition of convex set. From [8]:

  Definition 2. *Let V be a vector space over $\Re$. A subset $X \subset V$ is called convex if every line intersects X in an interval, that is,*

  $$(\lambda x_1 + (1 - \lambda)x_2) \in X$$

  when $0 < \lambda < 1$ and $x_1, x_2 \in X$.

  Therefore, the empty set is a convex set because there are no elements to draw any line, making it impossible to get a single line to which the inclusion condition is not verified. Boring enough, [15], p. 108, brings us the certainty that any intersection of convex sets is a convex set...

  If a convex set is supposed to contain all linear combinations of its points, as the definition last mentioned states, there is no doubt that if you take all linear combinations of the points of any given set E, you will have the smallest convex set containing E, is it not?

- Geometric shape intended: With Definition 2, the geometric intended shape is a continuous mass of points, with no allowance for 'holes', for otherwise one would be able to draw a straight line connecting points of the set in which line there would be a few (at least) members which would not be contained in the set under analysis, what would make of it a non-convex one, that is, concave.

- Criticisms:

– A definition, in Mathematics, that is, a creation axiom, should be written obeying the well-posedness guidelines. That would mean lowest number of words as possible, unique referents for World objects, or closest as possible to that, most objective wording as possible, but still complete, in the sense that any reader would be able to form same abstract idea (or even concrete) as that intended by creator of that axiom when simply reading it. An axiom is something unwanted in Mathematics, for it has got no proof. Therefore, its creation must be justified with top argumentation, and never include matters already dealt with in theorems, or other provable statements. Worst mistake of all in Definition 2 is, therefore, including the information 'the intersection of all convex sets containing E is itself a convex set'. Well-posedness really demands subtraction of that line from the axiom of creation, or definition, of the 'intended' mathematical entity 'hull';

– Hull is inadequate wording, as explained before, and should be immediately replaced by a word which satisfies the requirements for inclusion in the mathematical vocabulary. Obvious candidate is 'smallest convex set containing any set'. However, it is also possible to find the term 'convex cone' for the same sort of situation (see [13], for instance);

– The other point is about redundancy. If all which is created by Definition 2 is actually a set containing all basic members of a convex set, why bothering?

  Such set is trivially the smallest convex set containing E. Is there any doubts?

  One certainly does not need a tool as powerful as a creation axiom to make a simple observation from the definition axiom for convex sets...

  This should be stated as observation, remark, following the definition of convex sets, something like 'observe that if we make all linear combinations between the elements of a set A be contained in another set, C, C will trivially be the smallest convex set containing A'...

Final Diagnosis for Definition 2, 'hull':

The definition is redundant, absurd, and should not be a definition at all. Its status should be reduced to 'observation', or 'remark', if ever deserving any mathematical treatment: should simply appear right after the definition of convex sets appears stated. Mathematics is not about 'stars', or 'creating for creating', we really need to make use of the mathematical tools not for our own entertainment, pleasure, or occupation, but for the actual progress of human beings. If something is not suitable, we should really be dry and knock it out. This one is not suitable at all for Mathematics.

Presentation of Definition 3, 'hull':

Definition 3. *(alternative)The convex hull of any set S is the union of the convex spans of all the finite subsets of S.*

Analysis of Definition 3, 'hull':

- Background theory needed: A span is a 'brushing' over something. A convex span is then understood as being a 'convex brushing' over something. We may certainly broaden this understanding to grasp the idea of covering each subset of S with a convex set. A finite subset means that there is a countable number of elements to that set. The union of the spans means picking the set containing every, and each member, appearing in the conjunction of all spans. We have proven, in recent work, that a set cannot contain itself, ever (please refer to [11]). This way, a set is not its own subset. However, the set minus one element will always be. If we then take, as minimum reasoning work, the convex cover of the set minus one element, plus the convex cover of that one element, and join both, we could actually end up with a situation of disjoint sets, what could possibly pose a problem for this definition. Notice that there is no reference, in the definition, as to the set being continuous or not. However, in another subset, we could have that isolated element with another element of the set, and in creating the convex cover, we would have no holes left; as the result they intend is the union, we would definitely get a convex set as result (even because, as pointed out before, the union of convex sets is a convex set). However, we wonder if that is the smallest convex set containing S, once that seems to be the intention in the original definition of hull, to which this one seems to provide alternative wording. Unfortunately, the union will rarely provide us with smallest set. The union is no guarantee for not picking more than needed when joining sets, specially in a case for which all convex covers of lonely couple of elements may be disjoint.

- Geometric shape intended: What is sought here is definitely a convex 'envelope', or 'cover', for the set.
- Criticisms:
  - About the sigmatoid hull, we all now know of its obligation of 'giving up'...
  - The finite subsets is a cloudy and dangerous point in Definition 3. First of all, the reals is an infinite set, and it is convex, for it is not possible to join two points and get a hole...Yet, with finite subsets, one will never cover it, so that it would never have a hull...and that does not seem to be the intention of the 'trial of definition axiom';
  - As pointed out before, the union does not provide us with the smallest convex set, no guarantee... For it to be a needed axiom, once more, it has to be meaningful mathematically. Such is not: nothing special about it.

Final Diagnosis for Definition 3, 'hull':

This one looks more reasonable in its shape, but it is far more equivocated in its wording. There is absolutely no salvation for it. Better entirely forgotten. If anything, once more, all fixed, almost all wording, we then could have it as side remark to after the definition of convex sets.
We find a more exotic definition in [3]:

Presentation of Definition 4, 'hull':

Definition 4. *On an Euclidean space, of even dimension, we may introduce, by a choice of complex valued coordinate functions, $z_l, ..., z_n$, the structure of the complex n-space, C^n. We can then associate, with each compact subset, X, of our space, its polynomial convex hull, in C^n, denoted by $hull(X)$. By definition, $hull(X)$ is the set of all p in C^n, which satisfy the relation:*

$$|f(p)| \leq \max_{x \in X} |f(x)|,$$

for every polynomial $f(z_l, ..., z_n)$. When $X = hull(X)$, we say that X is polynomially convex in C^n.

Analysis of Definition 4, 'hull':

- Background theory needed: When the word 'space' gets mentioned in Mathematics, it refers to vector space. An Euclidean space bears complete metrics, that is, all foundational necessary axioms for a metrics to deserve being called a 'metrics', according to Euclid, are valid there. Basically, it is the place where Euclid would feel comfortable at (his axioms and theorems). That means one is able to calculate distance and work with the notion, all the way through, with no further problems. Dimension, in Mathematics, is equivalent to coordinate bearer universe. An even dimension would mean that every coordinate is 'married' to another, that is, they multiply themselves by couples...this is interesting and regards the geometric shape being nicer, once even numbers are nicer to deal with in Mathematics than odd. A compact set, which is complex, means the usual in the reals: limited and closed. However, in the reals, it is an interval, easy to deal with. In the complex plane, it has to be described graphically, or by means of reference to the real part of its members. A function, in Mathematics, is usually referred to as f, as most trivial internationally accepted notation, whilst a polynomial gets referred to as p. That is to make it easy and quick, well-posedness, so that any reader will remember first letter of the words, what works even in a psycholinguistics level. Therefore, when no need arises, when the function, or polynomial, being written about is only one, any mathematician has got, as minimum ethical obligation (help fellows to understand and progress further on non-reserved work), to use those letters in their scientific statements. This is a silent rule for Mathematics, made via habit and history, rather than via statement on its theory.

- Geometric shape intended: The geometric shape intended here is trivially the same as that intended by Definition 1.
- Criticisms:
  - The word hull should not be there, as explained before;
  - We cannot 'introduce' the structure of something which already exists in Mathematics. We may, at most, apply that structure replacing the name of the already existing coordinates;
  - It is either the case that they refer to polynomials or to functions. If polynomials, no sense calling them f, what creates unwanted psychological tie to functions, as explained earlier on in this very paper;
  - And if we choose functions, it should be more adequate to read 'functionally convex', rather than 'polynomially', if anything related;
  - Either it is the case that we refer to 'every polynomial', and therefore trivialize all to a single shape, or we refer to a specific result, attained in attachment to a single polynomial, and we refer to 'each chosen polynomial', instead;
  - A function might not be continuous and, therefore, possibly not allow the maximum to exist, so perhaps the words 'polynomial function' should be read there;
  - If it is the shape we are interested at, of course we should be considering not the complex number, but its image via polynomial function, for the calculation will generate any sort of graph if referring to the complex number instead;
  - The term 'convex' gets to be challenged here, once more. What

is it that is convex and gets formed via this axiom?

– The issue on $X = hull(X)$, also mentioned in [7], should be completely forgotten, for it is not true that the domain share is 'changed' from non-convex to convex via the definition created...For instance, the straight plane, line in the reals, is convex. Suppose we consider the horizontal plane generated by a limited and closed share of the complex numbers, then, as extension. What is missing to complete the graph is the rest, whole lot, that will be everything apart from a rectangle, for instance. A rectangle is a closed and limited share of the complex numbers set. However, that shape is not convex!

Final Diagnosis for Definition 4, 'hull':

It may be saved, just in the same sense the first definition may. However, serious fixing is required. If we take away the words 'convex', 'hull', and the 'introduction of the structure', as well as replace 'polynomial' with 'polynomial function', and 'for every' with 'for each chosen polynomial function', we may be able to get something more mathematically acceptable...
As for the old assertion, X =Hull(X) implying something, we suggest that, if anything at all, it implies $C_f^C = \emptyset$.

3 New Born Babies: Holder & Calyx

We wish for something born in the origin of the 'Infinitum world' (please refer to [10] for this matter) and dying by the own curve representing the function. This way, we believe not only we get the idea of 'bouquet', making it romantic and beautiful, but we also get the idea of 'support' for the 'line of the function'. And, on the top of that, not to state this is the most optimized way of making it be, we also get a path to pursue movement of randomly chosen image points, being able to finally work, by hand, the closest convex function curve, for instance, available. We are then interested in a straight line born at the origin of the 'Infinitum world', which is going to die at the precise image of the point, one for each function point, and attached to a particular member of the domain in a explicit manner.

We get the names 'holder' and 'calyx' for being them suitable for Mathematics lingo (both point uniquely to single reference in the actual World, one usually associated with things which will 'hold' the weight of something else, like balloons, etc, what makes it ideal thing to think of, 'holding the function in the air', other usually associated to the situation of the inverted strawberry (as from plant) hull - see [4] and [1], for instance). This way, we have:

HOLDER OF A FUNCTION

In the real numbers environment:

Definition 5. *Holder of a real function f, which we represent by $Holder(f(x))$, is:*

$$Holder(f(x)) =$$

$$\{g_i(x) \in X : g_i(x) = \frac{y_i}{x_i} x, \textit{if } x_i \neq 0 \veebar \ g_i(x) = y_i, \textit{if } x_i = 0 / (x_i; y_i) \in f \wedge i \in \Re_+\},$$

where X is the set of functions which either start or stop by the origin of the Infinitum World and hold, as other extreme, the ordered pair $(x_i; y_i)$.

In the complex numbers environment:

Definition 6. *Holder of a complex function f, which we represent by $Holder(f(z))$, is:*

$$Holder(f(z)) =$$

$$\{g_i(z) \in X : g_i(z) = \frac{z_{y_i}}{z_{x_i}} z, \textit{if } z_{x_i} \neq 0 \veebar \ g_i(x) = z_{y_i}, \textit{if } x_i = 0 / (z_{x_i}; z_{y_i}) \in f \wedge i \in \Re_+\},$$

where X is the set of functions which either start or stop by the origin of the Extended Infinitum World and hold, as other extreme, the ordered pair $(z_{x_i}; z_{y_i})$.

CALYX OF A FUNCTION

The idea of calyx came from trying to understand the idea of hull. Basically, what was thought of was the region either under or over the function graph, when the function is represented by its 'illusion' (modulus). It will obviously not work for concave functions, or convex graphs. We then wish to restrict the scope of functions passive of having a calyx associated to them to those with concave curves as representation, or their alike in spaces of more dimensions...

Notice that a function might be concave, for instance a parabola which 'looks down' in the Cartesian plane, and yet hold a convex function as 'illusion' of itself in the Cartesian plane, for if a parabola is drawn 'facing down' in the fourth quadrant of the Cartesian plane, for instance, when we pick our nice illusion from it, that is, change all its images to positive values, we will have same absolute size upwards, what will then finally give us a convex function...

We then make use of an odd expression, 'convex illusion', trying to express this idea. Of course, even our definition may be improved by others. We are obviously making use of the English language to express the precise idea of it, what is mathematically allowed and is actually obligation of everyone in Mathematics who is unable to find pointers in the language which could be candidates to belong to the mathematical lingo, that is, in not finding suitable mathematical lingo, one should definitely use as many English words as possible to make the reader see the same they see, or intend. That is not against the well-posedness, rather the opposite.

We also use the expression 'nice finishing' to mean a function which has got both first and last image at same height on the images axis, that is, if a plane is put on top of first and last image of the interval where the function domain set, a compact, is, one would then observe that plane is in perfect alignment with the horizontal ruler under it.

It was very complicated to devise a way of expressing the idea intended. Easy to refer to minimum image, but not to each point of the image...

Short Calyx

Definition 7. *Consider K, a compact subset of the complex plane. Consider f, a polynomial function, evaluated over K and with convex illusion of nice finishing. Consider its non-negative 'illusion', possibly coinciding with it, $|f(z)|$. Under the just mentioned conditions: SC_f is a process.*
SC_f holds two different sub-processes of formation:
1) $g(z_i) \in \mathcal{C} : z_i \in K, g(z_i) \geq 0$ will belong to SC_f if and only if $|g(z_i)| \leq |f(z_i)|$.

2) $g(z) \in \mathcal{C} : z_i \in (\mathcal{C} - K), g(z_i) \geq 0, |g(z)| \leq |f(z_0)|$*, where* z_0 *is the minimum of the domain compact set for* f *(or the maximum).*

Long Calyx

Definition 8. *Consider* K*, a compact subset of the complex plane. Consider* f*, a polynomial function, evaluated over* K *and with convex illusion of nice finishing. Consider its non-negative 'illusion', possibly coinciding with it,* $|f(z)|$*. Under the just mentioned conditions:* LC_f *is a process.*
LC_f *holds two different sub-processes of formation:*
1) $g(z_i) \in \mathcal{C} : z_i \in K$ *will belong to* SC_f *if and only if* $|g(z_i)| \leq |f(z_i)|$*.*
2) $g(z) \in \mathcal{C} : z_i \in (\mathcal{C} - K), |g(z)| \leq |f(z_0)|$*, where* z_0 *is the minimum of the domain compact set for* f *(or maximum).*

Remark 2. Of course the definitions for Calyx are automatically valid for the real numbers set as well ($b = 0$ for each $z \in \mathcal{C}$ represented as $a + bi$).

Remark 3. We also got to devise this weird entity of 'filling' of a function graph...by that meaning everything inside of a graph. For instance, if the curve is concave, we then get a set of points in the plane which could be 'inserted in its hole' to make of it a solid, instead of just a cask...it would be valid both for concave and convex curves...The names, if mathematical, and chosen according to the well-posedness, will allow the mathematician to refer to those regions of the plane in a universal manner when speaking to others, as minimum gain.

4 Fixed Old Trial of Definitions for Hull

From our Final Analysis for each definition here dealt with, only the first, which happens to equate to the fourth, in terms of introduction of entities, may be saved, if and only if it gets re-worded. This way, we here present the possible fixing for that one. Consider K as being any compact subset of the complex plane.

Definition 9. *Consider* f *as being a polynomial function with domain* $k \subset \mathcal{C}$ *and image in* $\mathcal{C}$*. Then:*

$$\hat{K} = \{z \in \mathcal{C} : |f(z)| \leq \max_{\zeta \in K} |f(\zeta)|\}.$$

On top of stating that $\hat{K}$ may now be renamed by anyone else (no inspiration whatsoever), we would like to create another concept, based in a few old ones:

Complement of a Graph

The complement of a graph of any complex function fully defined in the whole complex environment, C_f^C, is formed by all images of f attained via application of the complement set to the domain of f in relation to the complex numbers set to all the transformations included in f, that is:

Definition 10. *Provided a complex function is fully defined, for each and every complex number z, then:* $C_f^C = \{f(z), z \in (C - D_f)\}$.

5 Conclusions

In this paper, we have discussed the idea of 'hull' in depth, in a depth which was never found in the Mathematics literature so far. The gift of this paper is, once more, the expert holistic view over the forced matching process between Mathematics and real life language. We have now just tried to settle, for good, the matter of convex hulls. We discuss, as allurement, the idea of 'polynomially convex hull', as well as other infelicities appearing in the Mathematics scholarship for long. We have also proposed a few new definitions (Calyx, Holder, and Complement of a Graph), adding to the mathematical lingo. The conclusions were varied, from the non-sustainability of the term hull as part of the mathematical lingo, to the non-sustainability of the polynomial convex hull as sound candidate to a mathematical entity.

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