SHORT NOTE ON THE DEFINITION OF s_2-CONVEXITY

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Abstract. This is a research note on K_s^2. Basically, we have managed to find at least one group of counter-examples for K_s^2 whilst extension of K_1^2. As this besets the claims regarding K_s^2, we here produce fixing and further refinement on the primordial analytical definition of the class, according to the literature we had access to so far.

1. Introduction

As seen in [Pinheiro 2008], the determination of the functional class K_s^2 is provided by the following definitions:

Definition 1. *A function $f : X- > \Re$ is said to be s_2-convex if the inequality*

$$f(\lambda x + (1-\lambda)y) \leq \lambda^s f(x) + (1-\lambda)^s f(y)$$

holds $\forall \lambda \in [0,1]; \forall x, y \in X; X \subset \Re_+$.

Remark 1. If the complementary concept is verified, then f is said to be s_2-concave.

Criticisms:
We have recently found out a basic mistake involved in such definition, which, in principle, looks so tidy and remarkable.

As definitions are one of the most important elements of formal mathematics, we felt the need of producing a note solely on the topic.

Basically, there is an easy counter-example to the claim that K_s^2 extends K_1^2, which is $f(x) = -x$ and alike functions. We here then prove what perhaps does not need proof, that is, that this function is, indeed, besetting the acceptance of K_s^2 as an extension of Convexity, and we also propose due needed replacement for the definition of the classes of functions K_s^2.

2. Counter-examples

Suppose $f(x) = -x$. We then have the following sequence of contradiction with the definition of the class K_s^2:

$$f(\lambda x + (1-\lambda)y) \leq \lambda^s f(x) + (1-\lambda)^s f(y)$$

$$\iff -\lambda x - (1-\lambda)y \leq -\lambda^s x - (1-\lambda)^s y$$

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$$\iff \lambda x + (1-\lambda)y \geq \lambda^s x + (1-\lambda)^s y,$$

what is clearly untrue for $x, y \in \Re_+^*$.

Therefore, it can only be the case that the function $f(x) = -x$*, and all functions alike, will only be included in* K_s^2 *if changes are made to its definition.*

The note regards noticing that no function with non-negative domain, and not entirely null, will satisfy K_s^2 *pertinence requirements if* $|f(x)| \neq f(x)$.

3. Proposed fixing in the definition of K_s^2

Definition 2. *A function* $f : X- > \Re$*, where* $|f(x)| = f(x)$*, is told to belong to* K_s^2 *if the inequality*

$$f(\lambda x + (1-\lambda)y) \leq \lambda^s f(x) + (1-\lambda)^s f(y)$$

holds $\forall \lambda \in [0,1]; \forall x, y \in X; \forall 0 < s \leq 1; X \subset \Re_+$.
A function $f : X- > \Re$*, where* $|f(x)| = -f(x)$*, is told to belong to* K_s^2*, under the conditions above, if the inequality*

$$f(\lambda x + (1-\lambda)y) < \lambda^{\frac{1}{s}} f(x) + (1-\lambda)^{\frac{1}{s}} f(y)$$

holds $\forall \lambda \in [0,1]; \forall x, y \in X; \forall 0 < s \leq 1; X \subset \Re_+$.

Remark 2. If the complementary concept is verified, then f is said to be s_2-concave.

Remark 3. Notice now that, with this small fixing on the definition, we achieve the expected result: Whenever the value of the function does not equate its modulus result, we then having a negative image, we get a higher curve for limit by taking less negativity than we had originally in the function.

4. Conclusion

In this short note, we have produced needed fixing to the definition of the functional class K_s^2*. By now, several problems have ALSO been noticed by us regarding* K_s^1 *and we do intend to produce a note on that definition as well soon. We deal with* $S-$*convexity little by little from now onwards, due to both its relevance and controversial material up to 2001, when we started working with it.*

5. References

[Pinheiro 2008] M. R. Pinheiro. Convexity Secrets. Trafford Publishing. 2008. ISBN: 1425138217.

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