SHORT NOTE ON THE DEFINITION OF s_2-CONVEXITY

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Abstract. In this little note, we get to emphasize that our previous results, as for K_s^2, the little fixing on the definition of the class, implies taking for granted an omission of ours in our previous papers, and addition of results to fit the fixing. We also here make of K_s^2 the proper extension of Convexity in an official way, presenting proofs and everything else, also extending it to the whole set of real numbers.

1. Introduction

As seen in [Pinheiro 2008], the determination of the functional class K_s^2 is provided by the following definitions:

Definition 1. *A function $f : X- > \Re$ is said to be s_2-convex if the inequality*

$$f(\lambda x + (1-\lambda)y) \leq \lambda^s f(x) + (1-\lambda)^s f(y)$$

holds $\forall \lambda \in [0,1]; \forall x, y \in X; X \subset \Re_+$.

Remark 1. If the complementary concept is verified, then f is said to be s_2-concave.

Therefore, if we write about Convexity, sufficing making $s = 1$ on the above inequality, we get:

Definition 2. *A function $f : X- > \Re$ is said to be convex if the inequality*

$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda) f(y)$$

holds $\forall \lambda \in [0,1]; \forall x, y \in X; X \subset \Re_+$.

In recent work, we have managed to prove that the above definition of s_2-convexity implied inconsistency in Mathematics if not bearing a further restriction of f, which would be $|f(x)| = f(x)$.

Definition 3. *A function $f : X- > \Re$, where $|f(x)| = f(x)$, is told to belong to K_s^2 if the inequality*

$$f(\lambda x + (1-\lambda)y) \leq \lambda^s f(x) + (1-\lambda)^s f(y)$$

holds $\forall \lambda \in [0,1]; \forall x, y \in X; \forall 0 < s \leq 1; X \subset \Re_+$.
A function $f : X- > \Re$, where $|f(x)| = -f(x)$, is told to belong to K_s^2, under the conditions above, if the inequality

$$f(\lambda x + (1-\lambda)y) < \lambda^{\frac{1}{s}} f(x) + (1-\lambda)^{\frac{1}{s}} f(y)$$

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holds $\forall\lambda \in [0,1]; \forall x, y \in X; \forall 0 < s \leq 1; X \subset \Re_+$.

Remark 2. If the complementary concept is verified, then f is said to be s_2-concave.

In this paper, we wish to emphasize that the concept of convexity goes a bit beyond what is mentioned above: It also includes functions with any real domain.
And following this emphasis, we wish to finally make of K_s^2 *a class containing the entire set of convex functions, that is, a proper extension of* K_1^2.
In this paper, therefore, perhaps our most meaningful paper on S-convexity, we wish to promote K_s^2 *to proper extension of Convexity, both analytically and geometrically.*
We pursue the following sequence of presentation:

- K_s^2 *extends Convexity properly, both analytically and geometrically;*
- *Note on the results for* K_s^2 *obtained so far by us;*
- *Conclusion.*

2. Proper extension of Convexity

2.1. **Analytical proof.** *Suppose* f *is a convex function. Then*

$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda)f(y).$$

If $|f(x)| = f(x)$, *such implies*

$$f(\lambda x + (1-\lambda)y) \leq \lambda^s f(x) + (1-\lambda)^s f(y),$$

once both λ^s *and* $(1-\lambda)^s$ *are between* 0 *and* 1, *and so is* s. *If* $|f(x)| = -f(x)$, *it is true that*

$$\lambda f(x) + (1-\lambda)f(y) \leq \lambda^{\frac{1}{s}} f(x) + (1-\lambda)^{\frac{1}{s}} f(y)$$

instead, for there is less negativity to the right side of the inequality. Besides $s = 1$ *recovers Convexity, as wished for. Moreover, as* s *decreases, the value of the term to the right increases, also as wished for.*
As the only possible problem for the inequality is the sign of $f(x)$, *the domain of each function in* K_s^2 *may easily be extended to* $\Re$ *without difficulty.*

2.2. **Geometrical proof.** *Geometrically, the right side of the inequality for* K_s^2 *either overcomes or equates that of* K_1^2, *the continuous line formed, continuity solely depending of* $f(x)$ *being defined in the entire interval* $[f(x), f(y)]$, *has to rest above or over the geometric line used as limit in Convexity.*
Notice, as well, that $\lambda = 1$ *will produce* $f(x)$ *whilst* $\lambda = 0$ *will produce* $f(y)$. *As we get less from* $f(x)$, *we get more from* $f(y)$, *which is greater than* $f(x)$, *and before* $\lambda = 0.5$ *we get negativity winning, after we get positivity (derivative). This way, we actually hold a maximum on that value of* λ. *As* λ^s *is an exponential function, we should have a perfect shape, smooth, between those values. The maximum of the function trivially happens at* 0.5^s.

3. Conclusion

In this short note, we have proved that K_s^2 is a perfect extension of the concept of Convexity, both analytically and geometrically. Besides, we acknowledge the realization that we should write a paper with a summary of results, so far obtained by us for S-convex functions, in order to add the restriction regarding the moduli and to have the second part of the definition producing results as well. We should then be explaining that all our results so far should be regarded as containing a basic enthymeme, $|f(x)| = f(x)$. The new revisions in the definition of K_s^2 imply us having to state that:

Definition 4. *A function $f : \Re- > \Re$, where $|f(x)| = f(x)$, is told to belong to K_s^2 if the inequality*

$$f(\lambda x + (1-\lambda)y) \leq \lambda^s f(x) + (1-\lambda)^s f(y)$$

holds $\forall\lambda \in [0,1]; \forall x, y \in X; \forall 0 < s \leq 1$.
A function $f : \Re- > \Re$, where $|f(x)| = -f(x)$, is told to belong to K_s^2, under the conditions above, if the inequality

$$f(\lambda x + (1-\lambda)y) < \lambda^{\frac{1}{s}} f(x) + (1-\lambda)^{\frac{1}{s}} f(y)$$

holds $\forall\lambda \in [0,1]; \forall x, y \in X; \forall 0 < s \leq 1$.

Remark 3. If the complementary concept is verified, then f is said to be s_2-concave.

4. References

[Pinheiro 2008] M. R. Pinheiro. Convexity Secrets. Trafford Publishing. 2008. ISBN: 1425138217.

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