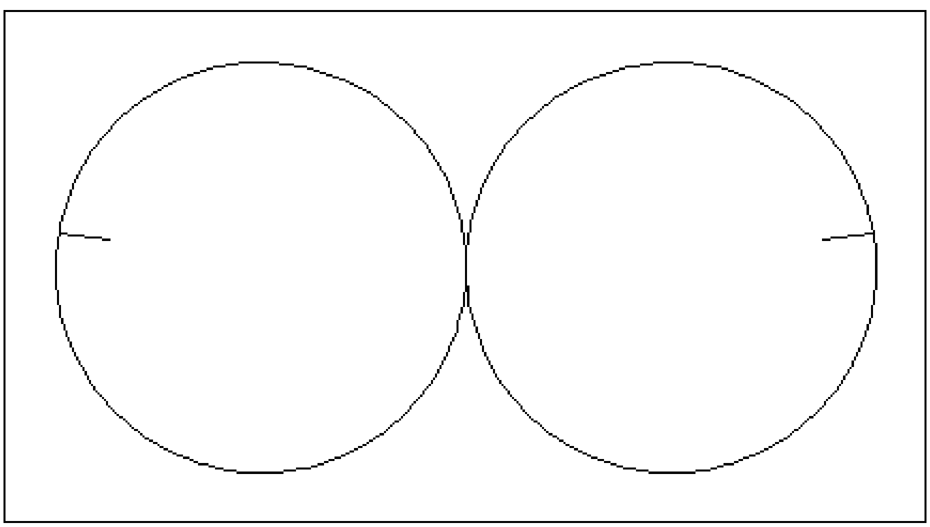
Gearing: Introduction.

The purpose of this essay is to explain to readers why clock and watch gears are designed as they are. It addresses the criteria that must be considered when designing a gear tooth.

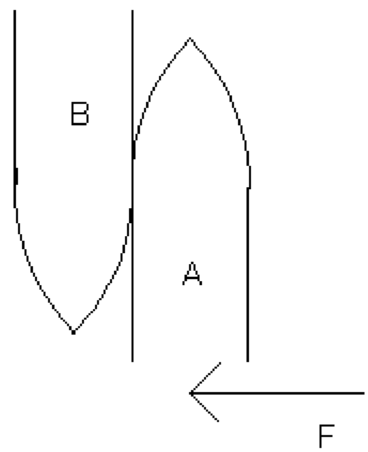
When a gear tooth engages a pinion leaf (tooth), it pushes the pinion leaf in its own direction of rotation, thereby transferring power to the pinion. The direction of the force that acts upon the pinion leaf could be seen as a tangent line on the edge of the gear tooth's circle (also called the pitch circle). This is similar to the action of the escape wheel on a pallet: see Chapter 4 of "Clock and Watch Escapement Mechanics."

In order to minimize power losses, we want the gear and pinion teeth to roll together as smoothly as possible, as when two well-honed disks roll together. The design of the gear and pinion teeth must be designed to simulate the rolling action as closely as possible to maximize the efficiency of power transferred from gear to pinion.

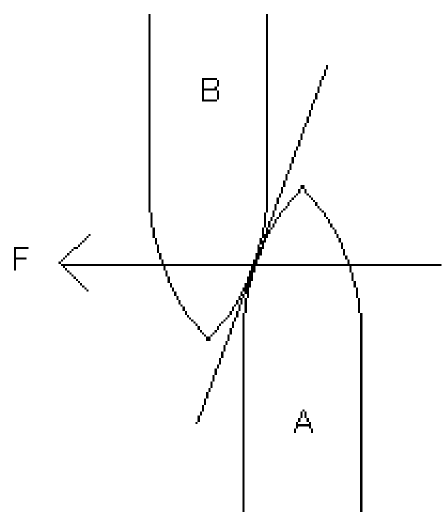


The ratio of the diameters of the two gears must be the same as the ratio of the number of teeth of the two gears. For example, if one gear has a diameter of 12 cm. and 120 teeth, and the other gear has a diameter of 1 cm., the other gear should have 10 teeth because the ratios are 12:1.

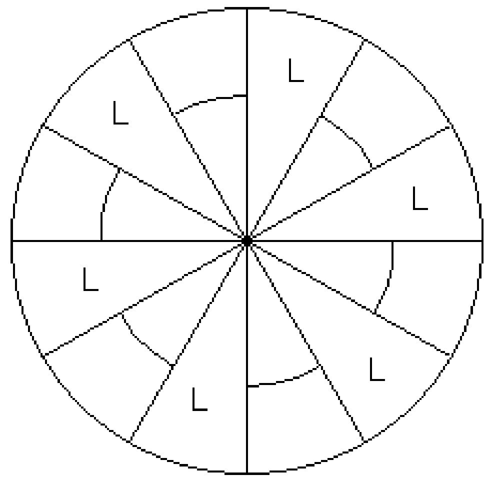
At the midpoint of the impulse, the point of contact between the gear tooth and pinion leaf must be on a part of the gear tooth (A) that is at right angles to the direction of the force that the gear tooth applies to the pinion leaf (B).



If the point of contact is not at right angles to the force, there is a loss of power. For example, if the depthing were too shallow, with the result that the point of contact were to be at 70° to the direction of force (instead of 90°), the efficiency loss due to vector forces would be about 12%, so the pinion would receive about 88% of the power from the gear. In addition to this, the direction of the force would result in some repulsion, causing undue wear of the bearings (or bushings).

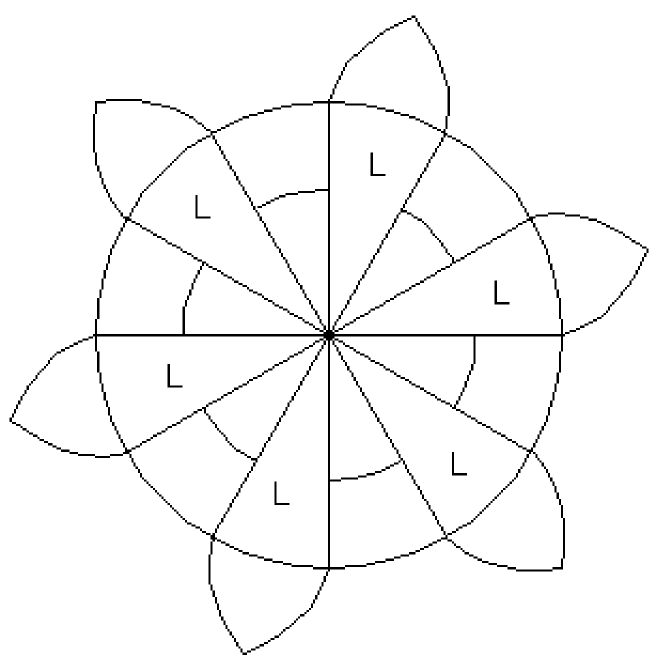


Since the point of contact must be at right angles to the direction of force, and the direction of force could be seen as acting at a tangent to the circumference of the gear's circle, so the lower part of the tooth (called the "dedendum") must be parallel to the radius line going from the circle center to the point of contact. Since the gear's circle is relatively large, the lines along the sides of the tooth's lower part (the dedendum) appear to be parallel, but they are not: they point to the gear's circle center. This is more obvious when you observe the dedendum of the pinion leaf, which appears to be tapered inwards, but both are designed on the same principle.



This drawing represents the basic design of the pinion leaf's dedendum, where the outer circle is the pitch circle, and each area labeled with an "L" represents the area occupied by a pinion leaf. The dedendum is the side of the leaf between the outer circle and the inner circle (shown here as parts of the circle).

The addendum is the part of the leaf that extends beyond the pitch circle. Each side of the addendum is a mirror image of the other side. (The addendum in this drawing is sketched only approximately).



The most important factor to consider is the angle of rotation of the two gears during the engagement of each tooth. Since the angle of rotation is greatest for the smaller gear (pinion), most attention is paid here. As the gear pushes the pinion leaf beyond the mid-point of the impulse, there are power losses caused by vector forces (the direction of the impulse is not the same as the direction of the pinion leaf receiving the impulse, so only a percentage of the impulse is received). If the angle between the two directions were small, the power losses would be small. The more teeth there are on the smaller gear, the smaller the angle of rotation of the two gears during the engagement of each tooth. This should be quite obvious since the angle occupied by each tooth in a 6 tooth pinion is 60º, whereas it is only 30° in a 12 tooth pinion (minus one degree to avoid binding). A 12 tooth pinion is much more efficient. A 12 tooth pinion is also much stronger because two pinion leaves are engaged at all times (the mid-point of engagement of the next leaf is reached before the first leaf is released).

The engagement of two gear teeth should be seen as two stages, engaging and disengaging. There is much more friction and power loss during engagement than during disengagement, so the teeth must be designed such that the tooth of the smaller gear (the pinion) is not released until the next tooth has reached the mid-point of the impulse. This is why the teeth of the larger gear extend beyond the pitch circle.

Take a moment to consider the effect of releasing the tooth of the smaller gear before the next tooth has reached the mid-point of the impulse. The next tooth is receiving the force of the tooth of the larger gear as it engages more deeply into the larger tooth, having a repelling effect and a grinding effect (the gear teeth are grinding into one another). The result of this can readily be seen in clocks with lantern pinions that have a bent or damaged pinion wires, or that have an unevenly spaced lantern pinion assembly (which may not be visible to the eye!): the great wheel gears of many American clocks that have lantern pinions have severe (abnormal) wear in the gear teeth. This severe wear is not caused by a weakness of the metal in the gear nor by the fact that lantern pinions are being used in the clock, but caused by a defect in the lantern pinion of the second wheel. If there is no defect visible in the lantern pinion wires, then a new lantern pinion assembly should be made for the second wheel as well as the great wheel being replaced.

You may have decided by now that the thing to do is to design the pinion with 12 teeth or more and to design the dedendum part of each tooth (the "flank") along the radial lines of the gear circle. Since the angle of engagement of each tooth would be very small, the design of the rest of the tooth would be less important and so the addendum could simply be rounded off, right? Almost, but not quite because of one more principle: during impulse, the angle of rotation of each gear must be proportional to the ratio of the teeth of each gear at each moment in time. If the gear providing the impulse (the "driver") rotates a little more in one instant that in another, while the rate of rotation of the pinion remains constant, then the rate of power transferred from the driver gear to the receiving gear would not be constant because work done is defined as the product of force and displacement (in this case, angle rotated). If the rotation of the pair of gears is not smooth and continuously proportional, the power transferred will not be even over the length of the impulse. In order to achieve this smooth transfer of power, the design of the addendum of each tooth must be such as to simulate the rolling action of two discs. When the gear teeth are designed so as to produce a constant angular-velocity ratio during meshing, they are said to have conjugate action.

Mathematicians have determined that this is best achieved by considering the path traced by a point on a circle that rotates on a flat plane (the Cycloidal Curve) and also on a curved plane (the Epicycloidal Curve).

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