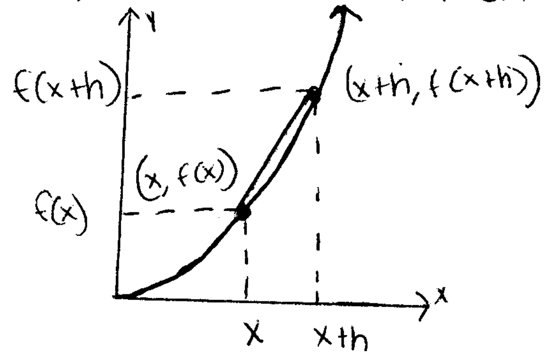
Math Analysis A Handwritten Quiz

Given: $f(x) = x^2 + x$, $x = 2$

1. Graph for the Definition of a derivative:



2. Deriving the slope of a tangent line to the curve:

$$\text{slope} = m = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

slope of a secant:

$$f(x) = \frac{f(x+h) - f(x)}{x+h-x}$$

Slope of a tangent to the curve is:

$$f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{x+h-x}$$

$f'(x)$ = slope of a tangent line to the curve:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{x+h-x}$$

3. Use Definition of the derivative to find slope.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{\cancel{x}+h-x}$$

Given $= x^2 + x$

$(x+h)(x+h)$

$x^2 + 2xh + h^2$

$$\lim_{h \to 0} \frac{(x+h)^2 + (x+h) - x^2 + x}{\cancel{x}+h-\cancel{x}}$$

$$\lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{x} + h - \cancel{x^2} - \cancel{x}}{h}$$

$$\lim_{h \to 0} \frac{2xh + h^2 + h}{h} \quad \frac{1h}{1h} = 1$$

$$\lim_{h \to 0} \frac{\cancel{h}(2x+h+1)}{\cancel{h}}$$

$$\lim_{h \to 0} 2x+h+1$$

$2x + (0) + 1$

$\boxed{f'(x) = 2x+1}$

This represents the slope of a tangent line to the curve or the derivative process

4. Power rule

$f'(x) = 2x+1$

Given $= x^2 + x$

$x^{2-1} + 1x^{1-1}$

$2x^{2-1} + 1x$

$2x^1 + 1x^0 = \boxed{2x+1}$ ✓

5. Find equation of a line tangent to the curve at the given location, as well as the slope intercept and standard form.

Given $x^2 + x$ $\quad x = 2$

$2^2 + 2$ $\quad (2, 6)$

$4 + 2 = 6$

This is the coordinate for the equation of the line tangent to the curve.

Slope:

$2x + 1$ $x = (2)$

$2(2) + 1$

$4 + 1 = \boxed{5}$ slope.

$y - y_1 = m(x - x_1)$

$y - 6 = 5(x - 2)$

$y - 6 = 5x - 10$

$+6 \qquad +6$

$\boxed{y = 5x - 4}$ slope-intercept form

$y - 6 = 5x - 10$

$-5x + y - 6 = -10$

$+6 \qquad +6$

$-(-5x + y = -4)$

$5x - y = 4$

Standard Form

6. Going from derivative back to the original function.

The constant C represents the constants which might have been lost in the derivative process

$$F(x) = \frac{x^{n+1}}{n+1} + C$$

$f'(x) = 2x + 1$

Given: $f(x) = x^2 + x$

$$\frac{2x^{1+1}}{1+1} + C = \frac{\cancel{2}x^2}{\cancel{2}} = \boxed{x^2 + x + C}$$