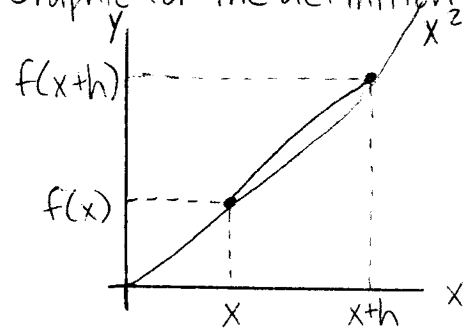
Math Analysis A Handwritten Quiz

Given: $f(x) = x^2 + x + 3,\ x = -1$

1. Graphic for the definition of a derivative



2. Formula for the slope of a tangent line to the curve

$$\text{slope} = m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

slope of the secant is

$$\text{slope} = m = \frac{f(x+h) - f(x)}{x+h-x}$$

slope of tangent to the curve is

$$\text{slope} = m = \lim_{h\to 0} \frac{f(x+h) - f(x)}{x+h-x}$$

$f'(x)$ = slope of tangent line to curve

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{x+h-x}$$

3. Find the slope of the tangent line to the curve using the function

$$f(x) = x^2 + x + 3$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{x+h-x}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - (x^2+x+3)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^2 + (x+h) + 3 - x^2 - x - 3}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{x} + h + \cancel{3} - \cancel{x^2} - \cancel{x} - \cancel{3}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{2xh + h^2 + h}{h} = \frac{\cancel{h}(2x+h+1)}{\cancel{h}}$$

$$f'(x) = \lim_{h\to 0} 2x + h + 1$$

$$f'(x) = 2x + (0) + 1 = 2x + 1$$

$$\boxed{f'(x) = 2x + 1}$$

4. Power rule

$$f(x) = x^2 + x + 3$$

$$f'(x) = 2 \cdot x^{2-1} + 1 \cdot x^{1-1}$$

$$f'(x) = 2x + 1 \quad \checkmark$$

5. Equations of a line tangent to the curve at $x = -1$

$$f(x) = x^2 + x + 3$$

$$f(-1) = (-1)^2 + (-1) + 3$$

$f(-1) = 3$ intercepts: $(-1, 3)$

$$f'(x) = 2x + 1$$

$$f'(-1) = 2(-1) + 1$$

$$f'(-1) = m = -1$$

$$y_2 - y_1 = m(x_2 - x_1)$$

$$y - 3 = -1(x+1)$$

$$y - 3 = -x - 1$$

$$+3 \qquad +3$$

$\boxed{y = -x + 2}$ slope-intercept form

$$y = -x + 2$$

$$+x - 2 \qquad +x - 2$$

$\boxed{x + y - 2 = 0}$ standard form

6. Find the anti-derivative

$$f'(x) = 2x + 1$$

$$F(x) = \frac{x^{n+1}}{n+1} + C$$

$$F(x) = \frac{2x^{(1+1)}}{(1+1)} + 1x + C$$

$$F(x) = \frac{2x^2}{2} + x + C$$

$$\boxed{F(x) = x^2 + x + C}$$

The constant C represents any constants which might have been lost in the derivative process.