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Interesting Sequences of Algebra and Geometry

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1 Analyzing Sequences

This presentation discusses how sequences are important in the study of algebra and geometry. How do sequences pertain to the right triangle? the binomial formula? etc. A *sequence* should be well defined that is for a given value there corresponds a real number as the range of the given function. It can begin at any number, 0, 1, are the most common. It can be infinite or finite.

An *arithmetic sequence* is one in which the terms differ by a constant.

$$\begin{array}{ccccccccccc} 2, & & 5, & & 8, & & 11, & & 14, & & \ldots, \\ & 3 & & 3 & & 3 & & 3 & & 3 & , \end{array}$$

$$2,\ 5,\ 8,\ 11,\ 14,\ \ldots,\ 3n-1$$

A *geometric sequence* is one in which the terms are related by a common ratio.

$$\begin{array}{ccccccccccc} 2, & & 6, & & 18, & & 54, & & 162, & & 486\ldots, \\ & 4, & & 12, & & 36, & & 108 & & \ldots & \\ & & 8, & & 24, & & 72, & & \ldots & & \end{array}$$

$$2,\ 6,\ 18,\ 54,\ 162,\ \ldots,\ 2\cdot 3^{n-1}$$

Notice that the ratio of the differences will remain constant. This will continue forever if you continue finding the differences in a geometric sequence.

Sometimes sequences are defined by recursive relations in which the next term depends on the previous ones. It could be the last two terms, three terms, any number of terms.

Usually sequences are represented as

$$a_1,\ a_2,\ a_3,\ a_4,\ \ldots,\ a_n$$

From here, how can we determine the next term in the sequence. Theoretically the next term can be virtually any number that one can think of. However, there are some systematic ways of determining the next number(s) in the sequence. When we look at our sequence and the differences

$$
\begin{aligned}
& a_1,\ a_2,\ a_3, a_4, \ldots, a_n \\
\Delta_1 = a_2 - a_1,\ \Delta_2 = a_3 - a_2,\ \ldots,\ & \Delta_n = a_{n+1} - a_n, \\
\Delta_1^2 = \Delta_2 - \Delta_1,\ \Delta_2^2 = \Delta_3 - \Delta_2,\ \ldots,\ & \Delta_n^2 = \Delta_{n+1} - \Delta_n, \\
\Delta_1^3 = \Delta_2^2 - \Delta_1^2,\ \Delta_2^3 = \Delta_3^2 - \Delta_2^2,\ \ldots,\ & \Delta_n^3 = \Delta_{n+1}^2 - \Delta_n^2, \\
& \ldots,\ \ldots,\ \ldots, \\
\Delta_1^K = \Delta_2^K - \Delta_1^K,\ \Delta_2^K = \Delta_3^K - \Delta_2^K,\ \ldots,\ & \Delta_n^K = \Delta_{n+1}^K - \Delta_n^K,
\end{aligned}
$$

with Δ_k being the first level of the difference, Δ_k^2, the second level, Δ_k^3, the third level, and so on. If the differences in the first level are constant, the sequence is linear. If the differences in the second level are constant then the sequences is quadratic. Typically, in schools the sequences given to students are quadratic. This is known as the method of finite differences. Sometimes sequences are defined recursively like the famous fibonacci sequence

$$f_n = f_{n-1} + f_{n-1}, \quad f_0 = 1,\ f_1 = 1.$$

Sometimes in fibonacci $f_0 = 0$ depending what the user decides. Questioning the students what is the next term in this sequence, they can give us the answer quickly. However if we ask them what is the 50th term?, it will take them more time. Mathematicians found a way to find a general formula for the nth fibonacci number [1]

$$f_n = \frac{1}{\sqrt{5}} \left[\left(\frac{1+\sqrt{5}}{2} \right)^{n+1} + \left(\frac{1-\sqrt{5}}{2} \right)^{n+1} \right].$$

From sequences that we know like the square numbers, the cubed numbers, etc., we can derive new sequences by means of a transformation. Add or Subtract a constant from them and make our new sequence. We can then ask the students "what is the next term in this sequence, the 10th term". We can also show a recursive definition for the squares, and for the cubes.

$$
\begin{array}{c}
1,\quad 4,\quad 9,\quad 16,\quad 25,\quad 36,\ \ldots, \\
3,\quad 5,\quad 7,\quad 9,\quad 11,\ \ldots \\
2,\quad 2,\quad 2,\quad 2,\quad \ldots
\end{array}
$$

$$f(n) = n^2, \quad a_n = a_{n-1} + 2n - 1, \quad a_0 = 0,\ n \geq 1,$$

is a recursive definition for the squares. The cubes

$$
\begin{array}{c}
1,\quad 8,\quad 27,\quad 64,\quad 125,\quad 216,\quad \ldots, \\
7,\quad 19,\quad 37,\quad 61,\quad 91,\quad \ldots \\
12,\quad 18,\quad 24,\quad 30,\quad \ldots \\
6,\quad 6,\quad 6,\quad \ldots
\end{array}
$$

$$f(n) = n^3, \quad a_n = a_{n-1} + 3n(n-1) + 1, \quad a_0 = 0,\ n \geq 1,$$

for the cubes. How can we arrive at this formula?

$$\begin{aligned} a_1 &= 1, \\ a_2 &= 1 + 6(1) + 1 = 8, \\ a_3 &= 8 + 6(3) + 1 = 27, \\ a_4 &= 27 + 6(6) + 1 = 64, \\ a_5 &= 64 + 6(10) + 1 = 125, \\ a_6 &= 125 + 6(15) + 1 = 216, \text{ etc.} \end{aligned}$$

1, 3, 6, 10, 15, ..., are the triangular numbers for which we know the formula $\frac{n(n-1)}{2}$, we substitute and we get the previous recursive formula. We can ask the students to simplify an expression like

$$1 + 7(n-1) + 12\frac{(n-1)(n-2)}{2} + 6\frac{(n-1)(n-2)(n-3)}{6}$$

which is just n^3 (but the students do not know that)!

2 Pascal's Triangle

$$\begin{array}{c} 1 \\ 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \\ 1 \quad 4 \quad 6 \quad 4 \quad 1 \\ 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \\ 1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1 \\ 1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1 \\ 1 \quad 8 \quad 28 \quad 56 \quad 70 \quad 56 \quad 28 \quad 8 \quad 1 \\ 1 \quad 9 \quad 36 \quad 84 \quad 126 \quad 126 \quad 84 \quad 36 \quad 9 \quad 1 \\ \cdots \end{array}$$

This is Pascal's Triangle(M1645) [2]. It can be represented by

$$\binom{n}{0}, \binom{n}{1}, \binom{n}{2}, \ldots, \binom{n}{n-1}, \binom{n}{n}$$

where the binomial coefficients are defined by

$$\binom{n}{r} = \frac{n!}{n\,(n-r)!}.$$

This is how one calculates the binomial coefficients. For instance, the sequence 1, 3, 6, 10, 15, 21, 28, 36, ..., known as the triangular numbers have the formula

$$\frac{n(n-1)}{2}, \quad n \geq 2$$

which is the binomial coefficient for

$$\binom{n}{2}$$

and after substituting $n+1$ in place of n will give

$$\frac{1}{2}n^2 + \frac{1}{2}n$$

when expressed as a polynomial in n. Using Pascal's Triangle one can expand binomials fairly easily. Expand $(x+y)^3$

$$\begin{aligned}(x+y)^3 &= \binom{3}{0}x^3y^0 + \binom{3}{1}x^{3-1}y^1 + \binom{3}{2}x^{3-2}y^2 + \binom{3}{3}x^{3-3}y^3 \\ &= x^3 + 3x^2y + 3xy^2 + y^3.\end{aligned}$$

This problem is fairly routine and should present no problems at all. It will probably be more exciting to ask them to expand $(2x^2+3y^3)^5$. Pascal's triangle can be used for relatively small powers. If we ask students to find the 14th term in the expansion of $(3x^2+2y^3)^{25}$ by using Pascal's Triangle it will be difficult for them. They will be better served if they use the Binomial Formula.

Many sequences can be extracted from Pascal's Triangle. The tetrahedral numbers, the pentagonal numbers, etc. Their formulas can easily be obtained by using the binomial coefficients.

3 Sequences involving Right Triangles

3.1 Sequences in which the leg and hypotenuse differ by 1

An interesting problem studied by many mathematicians is find all integer sided right triangles in which one leg and the hypotenuse differ by 1. The first primitive pythagorean triple (the sides are relatively prime to each other) 3, 4, 5 satisfies this condition. Which others satisfy these conditions. The next pythagorean triples that meet this criterion are the 5, 12, 13 right triangle and the 7, 24, 25 right triangle. Can we find a pattern to these primitive pythagorean triples sequence.

$$\begin{aligned}3, 5, 7, \ldots &= 2n+1 \\ 4, 12, 24, \ldots &= 2n(n+1) = 2n^2 + 2n \\ 5, 13, 25, \ldots &= 2n(n+1) + 1 = 2n^2 + 2n + 1\end{aligned}$$

How do we come up with these formulas? The first one is simple but the next two are a little bit more difficult. One way we see these formulas is by using the previous way of finding pythagorean triples using the formulas

$$x = 2uv, \quad y = u^2 - v^2, \quad z = u^2 + v^2. \tag{1}$$

By letting $u = n+1$ and $v = n$, we sustitute in (1) and obtain

$$\begin{aligned} x &= 2(n+1)n, \quad y = (n+1)^2 - n^2, \quad z = (n+1)^2 + n^2 \\ x &= 2n(n+1), \quad y = n^2 + 2n + 1 - n^2, \quad z = n^2 + 2n + 1 + n^2 \\ x &= 2n^2 + 2n, \quad y = 2n + 1, \quad z = 2n^2 + 2n + 1. \end{aligned} \tag{2}$$

The pythagorean triples generated by this formula (2) attributed to Plato always guarantees primitive pythagorean triples.

The other way is to analyze the differences of the terms as follows

$$\begin{array}{ccccccccccccc} 1, && 5, && 13, && 25, && 41, && 61, && \ldots, \\ & 4, && 8, && 12, && 16, && 20, && \ldots & \\ && 4, && 4, && 4, && 4, && \ldots && \end{array}$$

We can see that in the second level the difference is a constant so the equation will be quadratic $(Ax^2 + Bx + C)$. Since the first term is 1 in the sequence we get

$$\begin{aligned} x &= 0, \quad C = 1 \\ x &= 1, \quad A + B + 1 = 5 \\ x &= 2, \quad 4A + 2B + 1 = 13. \end{aligned}$$

Simplifying this will give

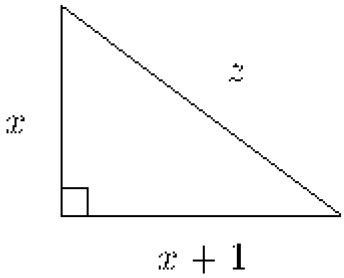
$$\begin{aligned} A + B &= 4 \\ 4A + 2B &= 12, \end{aligned}$$

for which $A = 2$ and $B = 2$. So the equation for the hypotenuse($h(n)$) will be $h(n) = 2n^2 + 2n + 1$. The largest leg is described by the sequence in which the leg is one less than the hypotenuse or $2n^2 + 2n$.

3.2 Sequences in which the legs differ by 1

We now want to find all integer sided right triangles in which the legs differ by 1, that is

$$\begin{aligned} x^2 + (x+1)^2 &= z^2 \\ x^2 + x^2 + 2x + 1 &= z^2 \\ 2x^2 + 2x + 1 &= z^2. \end{aligned} \tag{3}$$



When is Eq. (3) a square number? We can sustitute different values and we see that when $x = 3$, we get 25. By placing this equation into a spreadsheet or a little program we can crank out the values for which Eq.(3) is a square number. For $x = 3, 20, 119, 120, 696, \ldots$, (3) is a square number. Can we find a formula that will generate these values. We may begin by letting $x_1 = 3$, $x_2 = 20$, $x_3 = 119$, ... How can we get to 696?

$$\begin{array}{ccccccccccc} 3 & & 20 & & 119 & & 696 & & 4059, & & \ldots \\ & 17 & & 99 & & 577 & & 3363, & & \ldots & \\ & & 82 & & 478 & & 2786, & & \ldots & & \\ & & & & & \ldots & & & & & \end{array}$$

No pattern seems evident. However by putting myself to work I found a recursive formula that simply works!

$$x_n = 7(x_{n-1} - x_{n-2}) + x_{n-3}$$

For instance,

$$\begin{aligned} x_4 &= 7(x_3 - x_2) + x_1 \\ 696 &= 7(119 - 20) + 3. \end{aligned}$$

By using this formula we can find the rest of the numbers that satisfy (3). Can we find a formula that does not involve recursion? For the *Fibonacci sequence* one exists and for this one it can be found.

$$\begin{aligned} x_n - 7(x_{n-1} - x_{n-2}) - x_{n-3} &= 0, \text{ for } n \geq 3 \\ x^3 - 7x^2 + 7x - 1 &= 0 \end{aligned} \tag{4}$$

Now we need to find the roots of (4)

$$\begin{array}{r|rrrr} 1 & 1 & -7 & 7 & -1 \\ & & 1 & -6 & 1 \\ \hline & 1 & -6 & 1 & 0 \end{array}$$

$$x^2 - 6x + 1 = 0, \quad x = 3 \pm \sqrt{8}$$
$$r_1 = 1, \quad r_2 = 3 - 2\sqrt{2}, \quad r_3 = 3 + 2\sqrt{2}$$

The formula that we need takes the form

$$\begin{aligned} x_n &= c_1 r_1^n + c_2 r_2^n + c_3 r_3^n \\ x_n &= c_1 + c_2 \left(3 - 2\sqrt{2}\right)^n + c_2 \left(3 + 2\sqrt{2}\right)^n \end{aligned}$$

and from here we need to find c_1, c_2 and c_3,

$$\begin{aligned} c_1 + c_2 + c_3 &= 0, \\ c_1 + (3 - 2\sqrt{2})c_2 + (3 + 2\sqrt{2})c_3 &= 3, \\ c_1 + (3 - 2\sqrt{2})^2 c_2 + (3 + 2\sqrt{2})^2 c_3 &= 20. \end{aligned}$$

From here,

$$c_1 = -\frac{1}{2}, \quad c_2 = \frac{1 - \sqrt{2}}{4}, \quad c_3 = \frac{1 + \sqrt{2}}{4}$$

and

$$x_n = -\frac{1}{2} + \left(\frac{1 - \sqrt{2}}{4}\right)\left(3 - 2\sqrt{2}\right)^n + \left(\frac{1 + \sqrt{2}}{4}\right)\left(3 + 2\sqrt{2}\right)^n \tag{5}$$

Eq.(5) is a general formula and finds the smallest leg in the consecutive legs pythagorean triples sequence. The sequence for the hypotenuse is similar and may be obtained from Pell's numbers 1, 2, 5, 12, 29, 70, 169, 408, 985, ...

$$\begin{array}{c} 1, \quad 2, \quad 5, \quad 12, \quad 29, \quad 70, \quad 169, \quad 408, \quad 985, \quad \ldots \\ 1, \quad 3, \quad 7, \quad 17, \quad 41, \quad 99, \quad 239, \quad 577, \quad \ldots \\ 2, \quad 4, \quad 10, \quad 24, \quad 58, \quad 140, \quad 338, \quad \ldots \end{array}$$

Half of the 3rd level is same as the above numbers and denoted by

$$p_n = 2p_{n-1} + p_{n-2},$$

recursively or

$$p_n = \left(\frac{1}{2} - \frac{\sqrt{2}}{4}\right)\left(1 - \sqrt{2}\right)^n + \left(\frac{1}{2} + \frac{\sqrt{2}}{4}\right)\left(1 + \sqrt{2}\right)^n,$$

without recursion. It can be tried with a spreadsheet like Excel, QuatroPro, etc. entering

```
((1/2)-2^0.5/4)*(1-2^0.5)^A2+((1/2)+2^0.5/4)*(1+2^0.5)^A2
```

will generate the Pell's numbers. A subsequence of Pell's numbers p_n also yields the length of the hypotenuse of the right triangles with consecutive legs and the general formula can be found by letting n be $2n + 1$. Pell's numbers sustituted into Eq.(1) will generate the pythagorean triplets, $(3, 4, 5)$, $(20, 21, 29)$, $(119, 120, 169)$, $(696, 697, 985)$, ..., in which the legs differ by 1. These sequences are illustrated in [2] as (M3074, M3074+1, M3955).

4 Other Sequences

4.1 Sequences in Polygons

Finding the number of diagonals in an n-sided polygon. Clearly n has to be 3 or greater. Triangles, quadrilaterals, pentagons, hexagons, etc. A triangle has 0 diagonals, a quadrilateral has 2, a pentagon has 5, a hexagon has 9, and so on. The table

n	3	4	5	6	7	8	9	10	11	12	...
d	0	2	5	9	14	20	27	35	44	54	

shows the relationship and this exercise presents no difficulty for students. The general term will be

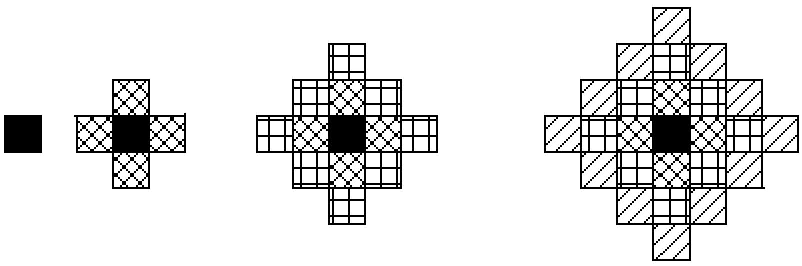
$$\frac{n(n-3)}{2}, \quad n \geq 3.$$

4.2 Sequences with blocks

By participating in the Eisenhower Grant for geometry under the leadership of Dr. Lee Von Kuster for Rio Grande City C.I.S.D., I was fortunate to learn about some interesting sequences that otherwise I would not have heard about. He gave us many problems with different situations some are hard to describe.

One of these situtations with blocks is as follows:

Given one block, enclose this blocks so that all corners are covered. How many blocks do we have? Now use more blocks to cover those blocks. How many blocks do we now have? How many blocks do we have at the 5th trial? How many blocks do we have in the nth trial?



1, 5, 13, 25, ...

This is essentially the same sequence as the lengths of the hypotenuse when the hypotenuse and the leg differ by one.

5 A stronger challenge

An interesting problem found on the ExCET preparation manual [4] will be a stronger challenge for the students. If we give this problem to the students, it is advised not to give the choices to answer the question. We can do better. We should ask the students to give us the general formula. Here's the problem:
A chemical engineer is developing a new adhesive. The amount of hardener added affects the adhesive's hardening time. The engineer has conducted five trials in which different amounts of hardener were added to the same amount of adhesive. If the pattern continues, which of the following would be the best estimate of the hardening time when 1 gram of hardener is used?

Trial	Amount of hardener (grams)	Time to harden
1	8	2 minutes
2	7	5 minutes
3	6	11 minutes
4	5	22 minutes
5	4	40 minutes

$$\begin{array}{c} 2 \quad 5 \quad 11 \quad 22 \quad 40, \ldots \\ 3 \quad 6 \quad 11 \quad 18, \ldots \\ 3 \quad 5 \quad 7, \ldots \\ 2 \quad 2, \ldots \end{array}$$

By looking at the subsequence we see that in the third level there is a constant difference.

5.1 Method 1

The equation will be a cubic equation $Ax^3 + Bx^2 + Cx + D = 2$,

$$\begin{aligned} x &= 0, \quad D = 2 \\ x &= 1, \quad A + B + C + 2 = 5 \\ x &= 2, \quad 8A + 4B + 2C + 2 = 11 \\ x &= 3, \quad 27A + 9B + 3C + 2 = 22 \end{aligned}$$

which can be simplified into

$$\begin{aligned} A + B + C &= 3 \\ 8A + 4B + 2C &= 9 \\ 27A + 9B + 3C &= 20 \end{aligned}$$

Find A, B and C from here and plug in back to the equation. Use calculator, Cramer's Rule, Elimination method, any method of your choice and obtain

$$a(n) = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{13}{6}n + 2, \quad n \geq 0$$

or

$$a(n) = \frac{2n^3 + 3n^2 + 13n + 12}{6}.$$

5.2 Method 2

We can find a formula right away by taking the first numbers in each level and multiplying them with their respective binomial coefficient for each level.

$$\begin{aligned}
2 + 3\binom{n}{1} + 3\binom{n}{2} + 2\binom{n}{3} &= 2 + 3n + 3\frac{n(n-1)}{2} + 2\frac{n(n-1)(n-2)}{6} \\
&= \frac{12}{6} + \frac{18n}{6} + \frac{9n^2 - 9n}{6} + \frac{2n(n^2 - 3n + 2)}{6} \\
&= \frac{12 + 18n + 9n^2 - 9n + 2n^3 - 6n^2 + 4n}{6} \\
&= \frac{2n^3 + 3n^2 + 13n + 12}{6}.
\end{aligned}$$

When n is 0 we get 2. So, this sequence is in terms of $n+1$ and we want it in terms of n, we substitute $n-1$ into

$$\begin{aligned}
a_n &= \frac{2(n-1)^3 + 3(n-1)^2 + 13(n-1) + 12}{6} \\
&= \frac{(n-1)\left[2(n-1)^2 + 3(n-1) + 13\right] + 12}{6} \\
&= \frac{(n-1)\left[2(n^2 - 2n + 1) + 3n - 3 + 13\right] + 12}{6} \\
&= \frac{(n-1)\left[2n^2 - 4n + 2 + 3n - 3 + 13\right] + 12}{6} \\
&= \frac{(n-1)\left[2n^2 - n + 12\right] + 12}{6} \\
&= \frac{\left[2n^3 - n^2 + 12n - 2n^2 + n - 12\right] + 12}{6} \\
a_n &= \frac{2n^3 - 3n^2 + 13n}{6}.
\end{aligned}$$

This is clearly the way to go instead of solving the 3×3 system of equations and finding the undetermined coefficients.

6 Just for fun

Practice multiplication facts by giving students problems like 142857×2, 142857×3, 142857×4, ..., till they get to 7. What happens?

12345679×9, 12345679×18, 12345679×27, 12345679×36, ... all the multiples of 9 till 12345679×81.

$11^0 = 1$, $11^1 = 11$, $11^2 = 121$, $11^3 =$? does the pattern continue or does it end?

$11^2 = 121$, $111^2 = 12321$, $1111^2 = 1234321$, $11111^2 =$?, will this pattern continue? or not?

If we ask the question "what is the next number in the sequence 1, 5, 13, ?", we can get different answers. Some will say that it will be 25 because

$$\begin{array}{ccccccc} 1, & & 5, & & 13, & & 25, \ldots, \\ & 4, & & 8, & & 12, & \ldots \\ & & 4, & & 4, & \ldots & \end{array}$$

and some could say 29 because

$$\begin{array}{ccccccc} 1, & & 5, & & 13, & & 29, \ldots, \\ & 4, & & 8, & & 16, & \ldots \\ & & 4, & & 8, & \ldots & \end{array}$$

This is why we have to be specific in how we define the sequence and how we ask the question. We need to give a little bit more terms so that students can guess the pattern. If

$$\begin{array}{ccccccccccccccc} 1, & & 5, & & 13, & & 29, & & 61, & & 125, & & 253, & & \ldots, \\ & 4, & & 8, & & 16, & & 32, & & 64, & & 128, & & \ldots & \\ & & 4, & & 8, & & 16, & & 32, & & 64, & & \ldots & & \end{array}$$

then, what is the general formula for this sequence?

7 Related TEKS

(a) (1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics. Students will continue to build on this foundation as they expand their understanding through other mathematical experiences. (This objective relates to Algebra I, Algebra II, and Geometry)

7.1 Algebra I

(b) (1) (B) The student gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities.

(b) (1) (C) The student describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations.

(b) (1) (E) The student interprets and makes inferences from functional relationships.

(b) (3) (A) The student uses symbols to represent unknowns and variables.

(b) (3) (B) Given situations, the student looks for patterns and represents generalizations algebraically.

Other (TEKS) could apply too.

7.2 Algebra II

(b) Foundations for functions: knowledge and skills and performance descriptions.

(b) (1) The student uses properties and attributes of functions and applies functions to problem situations. Following are performance descriptions.

(b) (1) (A) For a variety of situations, the student identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations.

(b) (1) (B) In solving problems, the student collects data and records results, organizes the data, makes scatterplots, fits the curves to the appropriate parent function, interprets the results, and proceeds to model, predict, and make decisions and critical judgments.

Other (TEKS) could apply too.

7.3 Geometry

(b) (3) The student understands the importance of logical reasoning, justification, and proof in mathematics.

(b) (3) (D) The student uses inductive reasoning to formulate a conjecture.

(b) (3) (E) The student uses deductive reasoning to prove a statement.

(b) (4) (c) Geometric patterns

(b) (4) (c) (1) The student uses numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures and solids, and angle relationships in polygons and circles.

(b) (4) (c) (3) The student identifies and applies patterns from right triangles to solve problems, including special right triangles (45-45-90 and 30-60-90) and triangles whose sides are Pythagorean triples.

Other (TEKS) could apply too.

7.4 Precalculus

(c) (4) The student uses sequences and series to represent, analyze, and solve real-life problems. The student is expected to:

(c) (4) (A) represent patterns using arithmetic and geometric sequences and series;

(c) (4) (B) use arithmetic, geometric, and other sequences and series to solve real-life problems;

(c) (4) (D) apply sequences and series to solve problems including sums and binomial expansion.

Other (TEKS) could apply too.

References

[1] A formula for the nth fibonacci numbers,
http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibFormula.html

[2] Sloane, N.J.A., Plouffe, S., Encyclopedia of Integer Sequences, Academic Press, 1995

[3] Sloane, N.J.A., "Online Encyclopedia of Integer Sequences",
http://www.research.att.com/~njas/sequences/

[4] Study Guide - Mathematics 17
http://www.excet.nesinc.com/excetstudyguid/17%20Mathematics.html

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