Coding against spreading gain optimisation of nonbinary BCH coded CDMA system

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The joint analytical optimisation of the spreading gain and coding gain of nonbinary BCH coded CDMA communication systems is considered in both single-cell and multi-cell scenarios. Furthermore, two types of detectors were employed, namely the minimum mean square error multiuser detector and the classic single-user matched filter detector. It is shown that the optimum coding rate varied over a wide range.

Introduction: When considering the CDMA system of Fig. 1, having a fixed bandwidth, the ratio of G/R_c has to be a constant, where G is the spreading gain and R_c is the channel coding rate. Therefore, it is possible to jointly optimise the spreading gain G and coding gain R_c. Recently, Yue and Wang [1] investigated the coding against spreading gain trade-offs in the context of a binary LDPC-coded CDMA system using BPSK modulation and simulations. By contrast, in this Letter we investigate the joint optimisation of the coding gain and spreading gain using nonbinary BCH codes in the context of a QPSK-modulated CDMA communication system by finding an analytical solution. In practice, the coding rate R_c assumes values of 1/2 or 1/3, while the spreading gain G may be in the range of 64 to 128. Hence in this Letter we consider a general system having a fixed ratio of $G/R_c = 256$, where QPSK modulation was invoked for communicating over a Rayleigh fading channel in both single-cell and multi-cell scenarios.

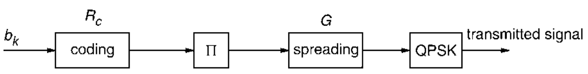

Fig. 1 *Schematic of transmitter*

Asymptotic performance analysis of CDMA: Tse and Hanly [2] investigated the asymptotic performance of CDMA systems, where the number of users K and the spreading gain G tend to infinity, and the ratio of $\alpha = K/G$ is fixed. Accordingly, the output signal-to-interference-plus-noise ratio (SINR) of the linear minimum mean square error (MMSE) multiuser detector (MUD) can be expressed as [2]:

$$\gamma_1 = \frac{P_1}{\sigma^2 + \alpha E_p[I(P, P_1, \gamma_1)]} \tag{1}$$

where

$$E_p[I(P, P_1, \gamma_1)] = \frac{PP_1}{P_1 + P_{\gamma_1}} \tag{2}$$

and P_1, γ_1 and P denote the power of the desired user, i.e user 1, the output SINR of the linear MMSE receiver for user 1 and the total power of the interfering users. Furthermore, Tse and Hanly [2] also indicated that the output SINR of the MF can be expressed as:

$$\gamma_1 = \frac{P_1}{\sigma^2 + 1/N\sum_{i=2}^{K} P_i} \tag{3}$$

In [3], Yu *et al.* extended this result to spectrally efficient M-ary quadrature amplitude modulation (QAM) communicating over the uncorrelated non-dispersive Rayleigh fading channel:

$$P_M = \left(1 - \frac{1}{M}\right) - 2\rho\left(1 - \frac{1}{M}\right) \times \left[\frac{1}{\sqrt{M}} + \frac{2}{\pi}\left(1 - \frac{1}{\sqrt{M}}\tan^{-1}\rho\right)\right] \tag{4}$$

where

$$\rho = \sqrt{\frac{\gamma_1}{2/3(M-1) + \gamma_1}}$$

Performance of nonbinary BCH codes: Nonbinary BCH codes [4] constitute an efficient class of linear codes and have the capability of correcting/detecting symbol errors. A BCH(N, Q) code has a total of N encoded symbols and Q original information symbols is capable of correcting up to $t = \lfloor (N-Q)/2 \rfloor$ random symbol errors and the corresponding coding rate is $R_c = Q/N$. In this Letter, we considered error-correction-only decoding [4]. According to [4], the decoded symbol error probability (SEP) after 'error-correction-only' decoding can be expressed as [4]:

$$P_s = \frac{n}{N}\sum_{n=\lfloor (N-Q)/2 \rfloor + 1}^{N} \binom{N}{n} (P_M)^n (1 - P_M)^{N-n} \tag{5}$$

where the Rayleigh channel's symbol error probability P_M before channel decoding is given by (4).

Numerical results: We considered both single-cell and multi-cell scenarios using pseudo-noise (PN) spreading codes. For the multi-cell scenario, we assume that each cell is surrounded by six adjacent cells having the same interference load. More explicitly, we considered a system supporting a total of $7K$ users having the power distribution of: $P_1 = P_2 \cdots = P_K = P$, $P_{K+1} = \cdots = P_{7K} = P = 12$. The classic Gaussian approximation of the interference was used. Again, we had a constant bandwidth associated with $G/R_c = 256$, while the channel was a single-path Rayleigh fading channel [4] and QPSK modulation was invoked in the system.

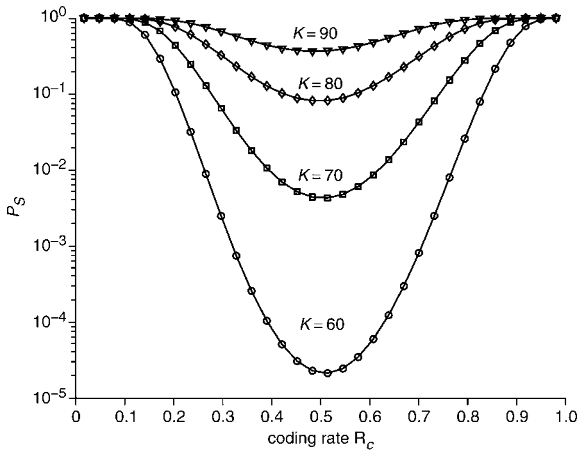

Fig. 2 *Symbol error probability (SEP) against coding gain at E_b/N_0 = 20 dB for K = 60, 70, 80 and 90 users and a constant bandwidth associated with $G/R_c = 256$, when communicating over uncorrelated non-dispersive Rayleigh channels in a multi-cell scenario*

$N = 256$, $Q = R_c N$

Fig. 2 portrays the symbol error probability (SEP) characterising the joint optimum of the coding gain and spreading gain, when communicating over an uncorrelated Rayleigh channel in an multi-cell scenario, where an MMSE MUD was invoked at the receiver. The number of users supported was varied from $K = 60$ to $K = 90$ and the $E_b = N_0$ value in the reference cell was 20 dB. From this Figure we can observe that the optimum coding rate in this scenario was $R_c = 1/2$.

To further characterise the system, Fig. 3 illustrates the optimum coding rate R_c against the number of users supported, when invoking different receivers in both single- and multi-cell environments. These results were computed from (5). Note however that at a higher user load the SEP is inevitably higher, despite using the optimum G and R_c values. Observe in Fig. 3 that the optimum coding rate for the matched filter (MF) detector varied between 0.3 and 0.2 in both the single-cell and multi-cell environments. However, for the MMSE MUD communicating in a single-cell environment we can observe that the optimum coding rate R_c is increased from 0.22 to 0.7, as the number of users supported is increased from 10 to 200. By contrast, for the MMSE MUD operating in a multi-cell environment, we can observe that the optimum coding rate R_c varied from 0.3 to 0.5 when the number of user supported was between 10 and 200.

Finally, Fig. 4 illustrates the optimum coding rate R_c against $E_b = N_0$, when invoking an MF and an MMSE MUD operating in both the single-cell and multi-cell environments, while the number of users supported was $K = 40$. From this Figure we may conclude that for the MF detector, the optimum code rate is approximately 1/3, while for the MMSE detector, optimum range is between 0.3 and 0.5.

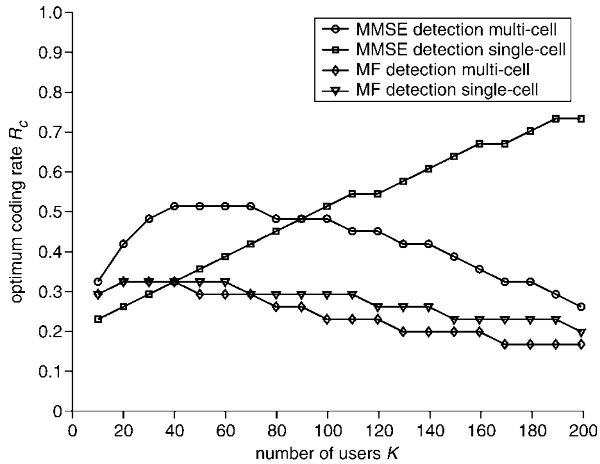

Fig. 3 *Optimum coding rate R_c evaluated from (5) against number of users supported, when invoking the single user MF and MMSE MUD receivers in both single- and multi-cell environments at $E_b/N_0 = 20$ dB*

An uncorrelated Rayleigh channel was used. $N = 256$, $Q = R_cN$

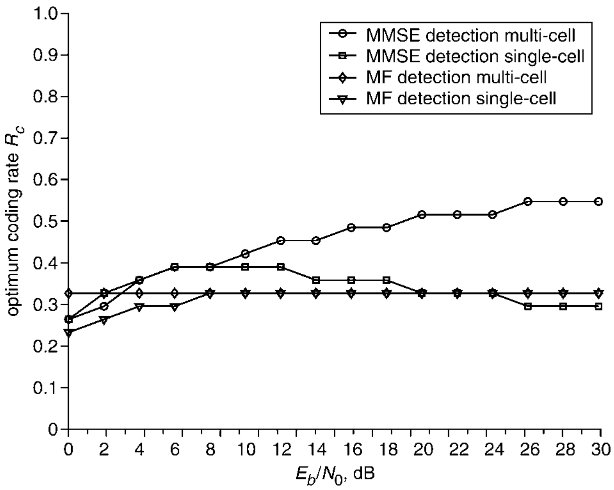

Fig. 4 *Optimum coding rate R_c evaluated from (5) against E_b/N_0, when invoking single user MF and MMSE MUD receivers in both single- and multi-cell environments at $K = 40$*

An uncorrelated Rayleigh channel was used. $N = 256$, $Q = R_cN$

Conclusions: In this Letter, we investigated the coding against spreading gain trade-offs in a nonbinary BCH coded CDMA system. The optimum coding rate R_c of the MF receiver is about 1/3, while that of the MMSE MUD ranges from 0.4 to 0.7, depending on both the user load K and the single against multi-cell scenario encountered.

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