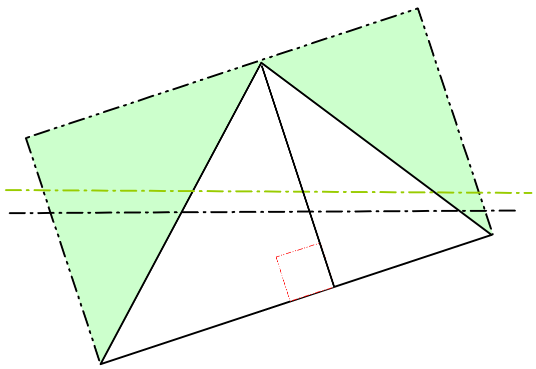
REVOLVED CIRCLE SECTIONS

Triangle revolved about its Centroid

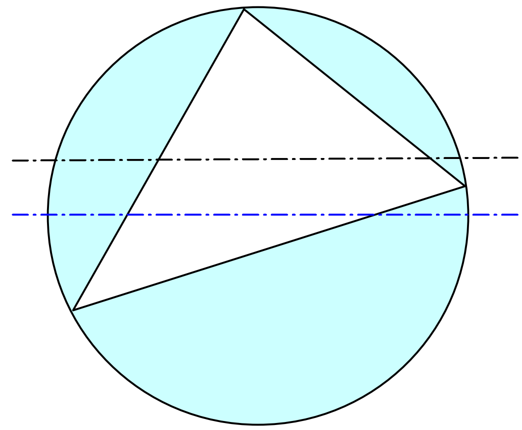
“Box-in” Method



Integrating to solve **I**, **Ax**, and **A** for a revolved triangle is difficult. A quadrilateral and another triangle are created above and below the centroid as the triangle in question is rotated.

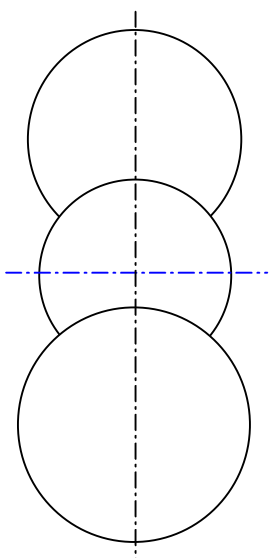
By drawing a rectangle around the triangle and dividing the figures along the altitude, the values of the right triangles with respect to the rectangles can be resolved.

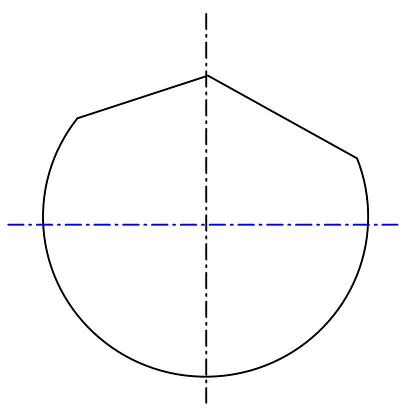
Circle Sector Method



The quantities **I**, **Ax**, and **A** required for engineering calculations may be determined for any circular sector (refer to **Circle Section Integrals**).

Subtracting the rotated sector values of interest from the circle values leaves only the triangle related quantities. The centroid of the triangle with respect to the circle center is easily calculated, making this the preferred method.

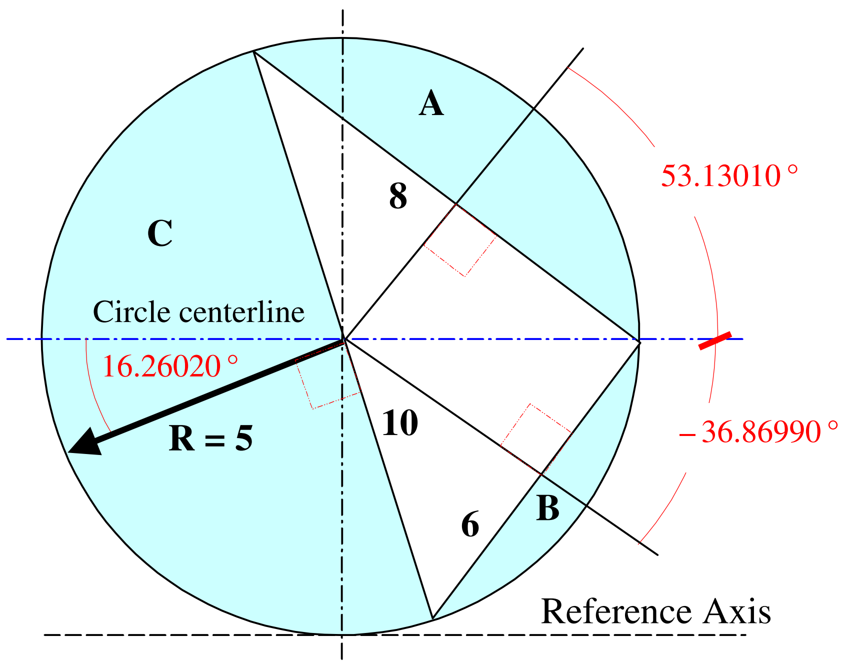




Maximum loads for multiple log beams with allowance for the scribe, or ridges with irregular pitches, may be estimated with relative ease using circle sectors.

REVOLVED CIRCLE SECTIONS

Triangle revolved about its Centroid
Sample Calculation



Sector A
A = 11.18238
x = 3.81553
r = 3.05242
I_X = 37.88988
I_Y = 3.09409
$I_R + Ar^2$ = 119.81007

Sector B
A = 4.08753
x = 4.40364
r = 2.64218
I_X = 7.54704
I_Y = .28153
$I_R + Ar^2$ = 33.46706

Sector C
A = 39.26991
x = 2.12207
r = .59418
I_X = 245.43693
I_Y = 68.59810
$I_R + Ar^2$ = 245.43698

Triangle
A = 24
h = 4.8
x = 1.6
r = zero

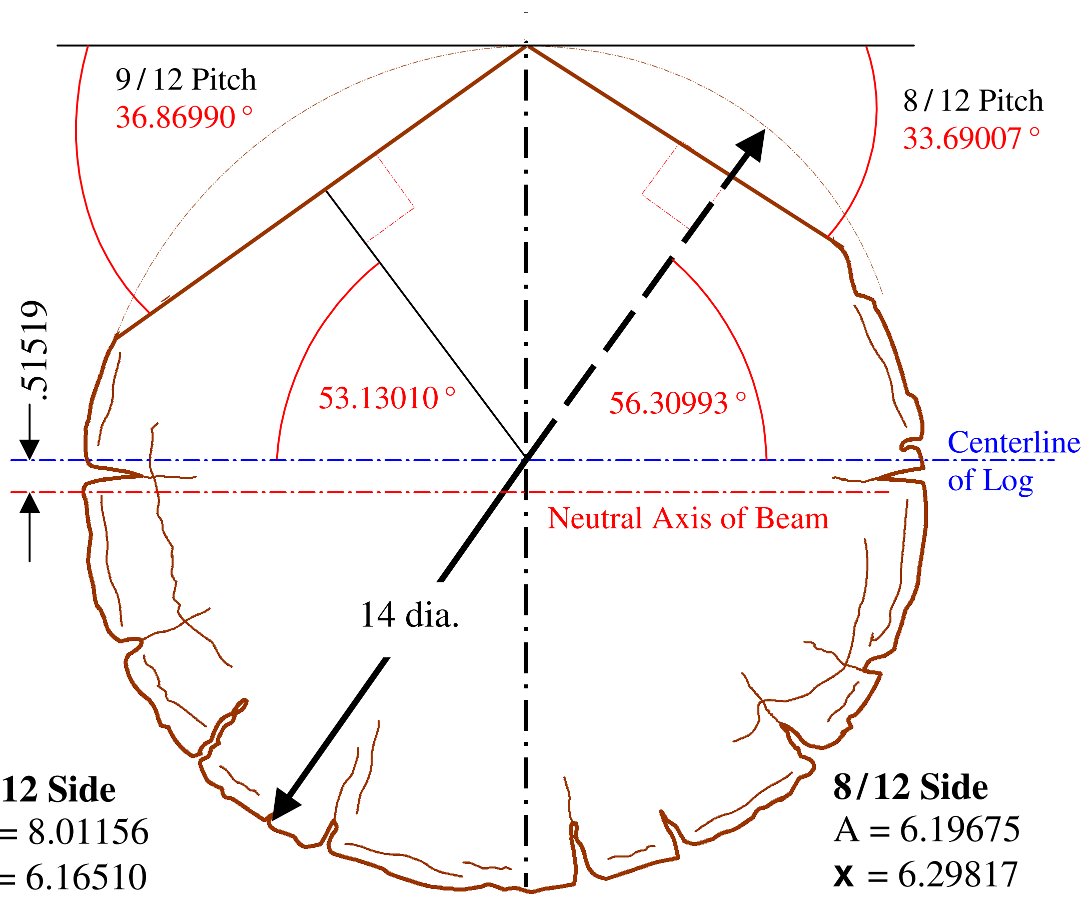
$\mathbf{I}_{\text{SEMI-CIRCLE}}$ = 245.43693
$\mathbf{I}_{\text{TRIANGLE}}$ = 245.43693 – (119.81007 + 33.46706)
= 92.1598

Note : Most calculations return $\mathbf{I}_{\text{TRIANGLE}} = I \pm Ar^2$ at this stage and require further work (see example below). The angle of revolution in this case places the median, and hence the centroid of the triangle, on the circle centerline. Thus, 92.16 is the final result.

Calculations with respect to the Reference Axis: $\mathbf{I} + \mathbf{Ar}^2$
$\mathbf{I}_{\text{CIRCLE}}$ = 2454.36926
$\mathbf{I}_A$ = 740.70277
$\mathbf{I}_B$ = 27.65541
$\mathbf{I}_C$ = 993.85073
$\mathbf{I}_{\text{TRIANGLE}} = \mathbf{I}_{\text{CIRCLE}} - (\mathbf{I}_A + \mathbf{I}_B + \mathbf{I}_C)$ = 692.16035
$\mathbf{I}_{\text{TRIANGLE}}$ about its centroid: $I - Ar^2 = 692.16053 - 24\,(5)^2 = 92.16053$

REVOLVED CIRCLE SECTIONS

Ridge Beam: Second Moment of Area
Sample Calculation



9 / 12 Side
A = 8.01156
x = 6.16510
r = 4.93208
I_X = 28.99274
I_Y = 1.08153
I_R = 11.12957
$I_R + Ar^2$ = 206.01407

Initial Log Calculations
A = 49π = 153.93804
$I = 7^4\pi/4$ = 1885.74099

8 / 12 Side
A = 6.19675
x = 6.29817
r = 5.24039
I_X = 1 9.08178
I_Y = .58949
I_R = 6.27943
$I_R + Ar^2$ = 176.45264

Final Beam Calculations: Log values – Sector values

$\mathbf{I}_{BEAM}$, wrt Log center = 1885.74099 – (176.45264 +206.01407) = 1503.27428

$\mathbf{A}_{BEAM}$ = 153.93804 – (6.19675 + 8.01156) = 139.72973

First Moment of Area = 0 – (32.47339 + 39.51365) = – 71.98704

Centroid = – 71.98704 ÷ 139.72972 = – .51519 (= 6.48481 from bottom)

$\mathbf{I}_{BEAM} = \mathbf{I} - (\mathbf{Ar}^2)$ = 1503.27428 – 139.72972 (– .51519)^2 = 1466.18711

Taking the bottom edge of the log as a reference line yields the same result:

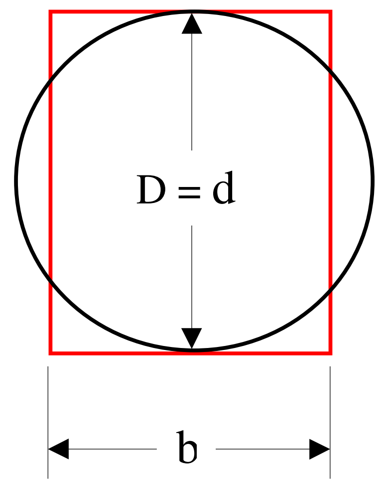
$\mathbf{I}_{LOG} - \mathbf{I}_{SECTORS}$ = 9428.70495 – (934.72081 + 1151.77168) = 7342.21246

$\mathbf{I}_{BEAM} = \mathbf{I} - (\mathbf{Ar}^2)$ = 7342.21246 – 139.72792 (6.48481)^2 = 1466.19198

RECTANGULAR SECTION
equivalent to a
CIRCULAR SECTION

Circle: $I = \dfrac{\pi D^4}{64}$

Rectangle: $I = \dfrac{bd^3}{12}$



Setting the values of I equal,
and the rectangle depth d = D:

$$\frac{\pi D^4}{64} = \frac{bd^3}{12} \text{, and } b = \frac{3\pi D}{16}$$

Circle: $A = \dfrac{\pi D^2}{4}$ Rectangle: $A = bd$

Substituting equivalent Circle values: Rectangle $A = \dfrac{3\pi D^2}{16}$

Equating the values of I with d = D ensures both beams have equal resisting moments **and** deflections.*
The distance from the neutral axis to the extreme fibre **c** is also equal for both sections.

These conditions are satisfied if $D = \sqrt[3]{\dfrac{32\mathbf{M}}{\pi \mathbf{F}_b}}$

Comparing the two areas, the rectangle area is only 3/4 of the area of the circle. The maximum shear stress of the circular section is 2/3 that of the equivalent rectangular section.

* For a uniformly loaded beam, supported at the ends:

$\mathbf{M}_r = \dfrac{F_b \mathbf{I}}{\mathbf{c}}$ Maximum deflection $= \dfrac{5w\boldsymbol{l}^4}{384\,\mathbf{EI}}$

ANALYSIS of LOG SPAN TABLES FORMULAS

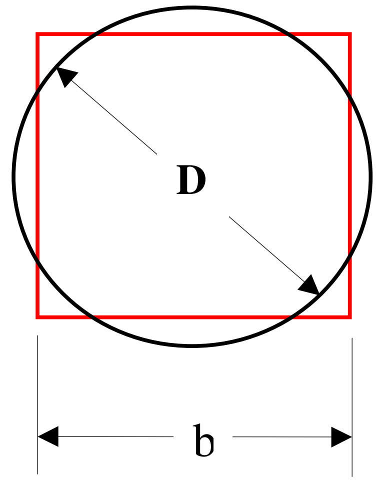
Log Diameter from Bending Moment Formula

Squaring the Circle

Circle: $A = \frac{\pi D^2}{4}$

Square: $A = b^2$

$\therefore b = \sqrt{\frac{\pi D^2}{4}}$



$$\sqrt{\frac{\pi D^2}{4}} = \sqrt[3]{\frac{6M}{F_b}}$$

The Bending Moment limit formula on page 47, with the terms re-arranged.

Section Modulus of a Square: $\mathbf{S} = \frac{b^3}{6}$, $\therefore M = F_b\mathbf{S} = \frac{F_b b^3}{6}$

$$b = \sqrt[3]{\frac{6M}{F_b}}$$

The formula for a beam of square section, equivalent to the formula on page 68 when **b** = **h**

Substituting $\sqrt{\frac{\pi D^2}{4}}$ for b gives the formula on page 47.

Section Modulus of a Circle: $\mathbf{S} = \frac{\pi D^3}{32}$, $\therefore M = F_b\mathbf{S} = \frac{F_b \pi D^3}{32}$

$$D = \sqrt[3]{\frac{32M}{\pi F_b}}$$

Solving for D yields the formula for a circular section based on the **section modulus**.

This equation disagrees with the Log Span Tables formula, which is based on the width **b** of a beam of **square section** that has an **area** equal to a circular cross section. The Log Span Tables version tends to return smaller log diameters. The following shear limit formula also compares cross sectional areas.

ANALYSIS of LOG SPAN TABLES FORMULAS
Log Diameter from Shear Formula

Re-arranging the shear limit formula on page 49:

$\pi D^2 = \dfrac{6V}{F_v}$, and dividing both sides of the equation by 4 gives:

$\dfrac{\pi D^2}{4} = \dfrac{6V}{4F_v} = \dfrac{3V}{2F_v}$ The left hand term is the area of a circle.

Reference for the following formulas: **Mechanics of Materials**, J. Lister Robinson, John Wiley & Sons, 1967, pages 96-97

Re-arranging the shear limit formula for a rectangle:

$F_v = \dfrac{3V}{2bh}$, $\therefore bh = \dfrac{3V}{2F_v}$ The variables **bh** on the left side equal the area of a rectangle.

$F_v = \dfrac{QV}{Ib}$ This is a general formula for maximum shear stress, ***regardless of the cross sectional geometry***.

Q is the First Moment of area above the Neutral Axis,
for a Circle: $Q = D^3 \div 12$ [Rectangle: $Q = \mathbf{bh}^2 \div 8$]
Circle width at the Neutral Axis: $\mathbf{b} = D$ [Rectangle: $\mathbf{b} = \mathbf{b}$]
Circle Second Moment of Area: $I = \pi D^4 \div 64$ [Rectangle: $I = \mathbf{bh}^3 \div 12$]

Substituting for the variables Q, **b**, and I for the **circle** in the general formula and solving for D gives:

$D = \sqrt{\dfrac{16\mathbf{V}}{3\pi F_v}}$ This equation differs from the formula in the Log Span Tables.

The Log Span Tables version returns larger log diameters. Substituting the appropriate rectangle variables in the general formula returns the standard equation $\mathbf{F_v} = 3V \div 2\mathbf{bh}$, in accord with the Log Span Tables (page 70).

NOTES and REFERENCES

Since measurements are with respect to the center of a circle, no distiction is made between moment arms **x** and **y**
The quantity **r** denotes the moment arm **y** from the ***x*** - axis
$\mathbf{I}_X$: Second Momemt of Area about the ***x*** - axis
$\mathbf{I}_Y$: Second Momemt of Area about the ***y*** - axis
$\mathbf{I}_R$: Second Momemt of Area revolved, with respect to the ***x*** - axis

The **Integrator** at mathworld.wolfram.com returns the same integrals for **A**, **Ax**, and **I** as described in **Circle Section Integrals**.

The "Box-in" and "Circle Sector" methods of calculation both return equal values for **A**, **Ax** and **I** and polar moment **J** of a triangle.

Calculated centroids of triangles and log beam sections may be tested by making a lamina:
The lamina will balance horizontally or level about the centroid.
The lamina may be rotated to any position, provided the plane of the lamina is plumb, and the line of a plumb bob will pass through the centroid of the lamina.

Reasonably accurate estimates of **A**, **Ax**, **I**, and $\mathbf{F}_v$ for multiple log beams may be obtained by summation employing horizontal strips.

Engineering values for circles representing log cross sections are calculated on the basis of bending moment, deflection and shear stress, without reference to "equivalent" rectangular or square sections.

Log Span Tables
B. Allan Mackie, Norman A. Read, Thomas M. Hahney
2000 Canada, pages as referenced above

Mechanics of Materials
J. Lister Robinson, John Wiley & Sons, 1967, pages 96-97

Joe$^{-\pi i}$