Notes re: Kernels

Having determined Deck angles **DD** and **D**, the Main and Adjacent kernels may be separated along the Hip/Valley plane, and the remainder of their respective angles can be solved by treating each kernel as an independent entity.

Five angles are relevant: **SS**, **DD**, **R1**, **P2**, and **C5**, or, the **cognates** of these angles. The names of sets of angles may vary from one kernel to another, but the relationships between angles located in similar positions always remain the same.

For convenience, the "Hip Run" is set equal to one; this immediately results in lengths that are trig functions of the Deck angles, **DD** and **D**, and Hip/Valley angle **R1**.

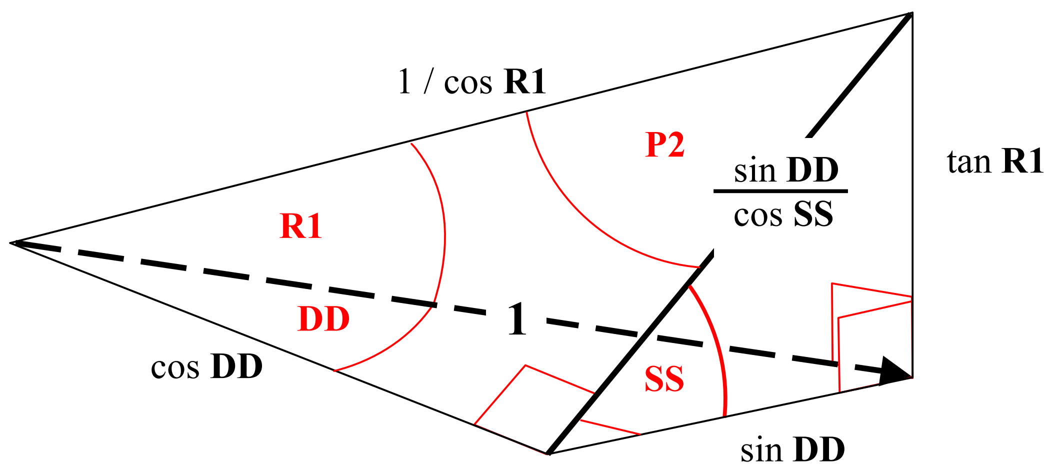
If the "Hip Run" does not equal one, all kernel dimensions still remain proportional. In fact, given any two angles and any one dimension, an entire kernel may be solved in terms of angles and lengths.

Kernels may be extracted directly from members such as hip rafters, common rafters, and purlins, as well as the overall roof. Tetrahedral kernels with right triangular faces are the simplest models to work with, but other theoretical geometries are possible.

Note that the diagram for Backing Angle **C5** actually consists of three interlocking kernels. Further angles, for example, **A7** and **P5**, may be defined, and relationships among both the "new" angles and angles previously defined may be resolved.

P2 ANGLE EQUATIONS:

Resolve the lengths of all sides of the kernel,
with respect to the unit radius vector.
Right angles are located as per previous diagrams.



$$\tan \mathbf{P2} = \frac{\cos \mathbf{DD}}{(\sin \mathbf{DD} / \cos \mathbf{SS})}$$

$$= \cos \mathbf{DD} \cos \mathbf{SS} / \sin \mathbf{DD}$$

$$= \cos \mathbf{SS} / \tan \mathbf{DD}$$

$$\sin \mathbf{P2} = \frac{\cos \mathbf{DD}}{(1 / \cos \mathbf{R1})} = \cos \mathbf{DD} \cos \mathbf{R1}$$

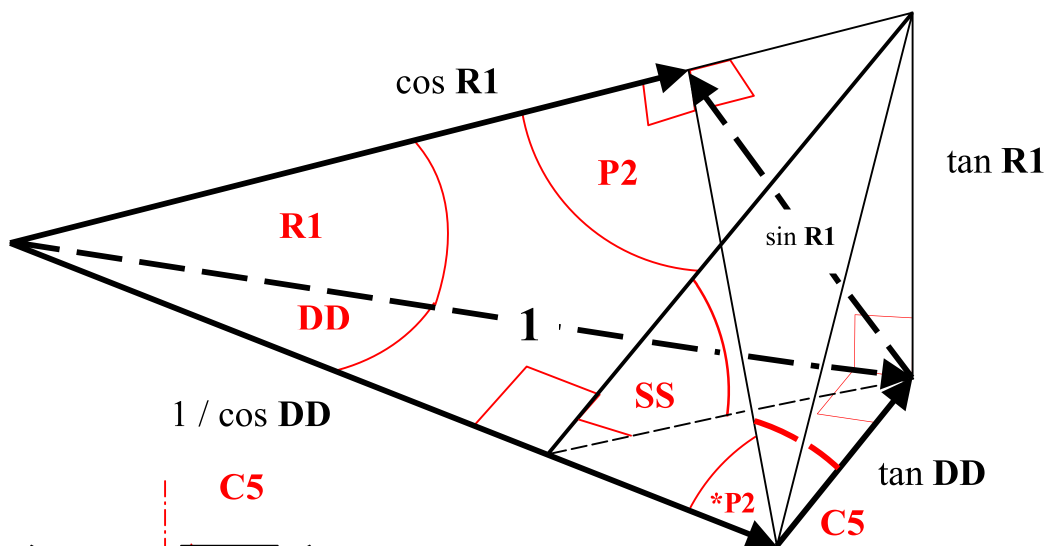
$$\cos \mathbf{P2} = \frac{(\sin \mathbf{DD} / \cos \mathbf{SS})}{(1 / \cos \mathbf{R1})}$$

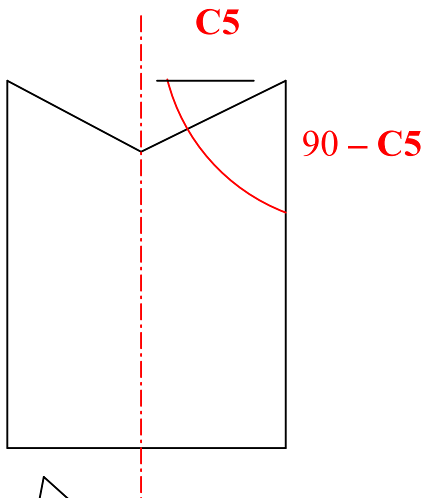
$$= \sin \mathbf{DD} \cos \mathbf{R1} / \cos \mathbf{SS}$$

This equation for cos **P2** may be reduced to a simpler formula in terms of two other angles. First, an analysis of the backing angle, **C5**, equation.

C5 ANGLE EQUATIONS:

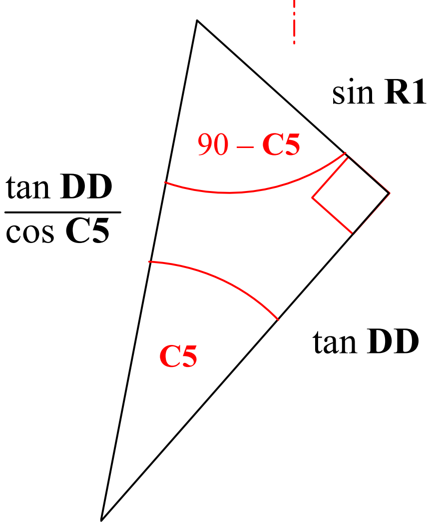
Lengths and angles not specifically shown on this drawing are as per diagram for **P2** angles.





The unit vector, tan **R1**, and tan **DD** lines are mutually perpendicular.

The dihedral angle between the plumb plane through the long axis of a hip or valley rafter, and the roof plane, is angle 90-**C5**.



tan **C5** = sin **R1** / tan **DD**

This is the simplest form of the equation for **C5**.

P2 and C5 ANGLE EQUATIONS:

Returning to the equations for angle **P2**; note the two locations of this angle on the roof plane:

$$*\cos \mathbf{P2} = \frac{(\tan \mathbf{DD} / \cos \mathbf{C5})}{(1 / \cos \mathbf{DD})} = \frac{\tan \mathbf{DD} \cos \mathbf{DD}}{\cos \mathbf{C5}} = \sin \mathbf{DD} / \cos \mathbf{C5}$$

Re-arranging the terms: cos **C5** = sin **DD** / cos **P2**

Recall that cos **P2** = (sin **DD** cos **R1**) / cos **SS**

Setting the cos **P2** formulas equal to each other:
sin **DD** / cos **C5** = (sin **DD** cos **R1**) / cos **SS,**
and 1 / cos **C5** = cos **R1** / cos **SS**

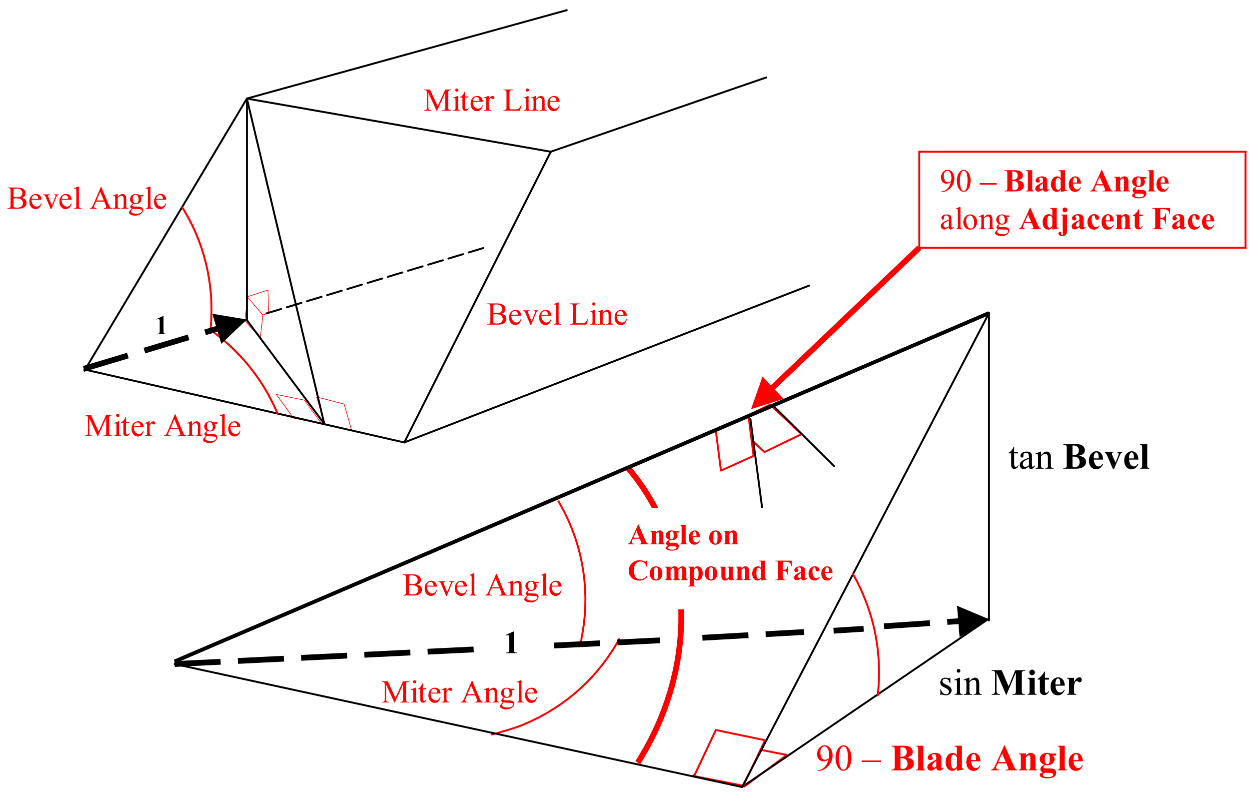
Therefore, cos **C5** = cos **SS** / cos **R1,** a formula for sizing valleys to common rafters.

Since sin **C5** = tan **C5** cos **C5**,
Substituting: sin **C5** = (sin **R1** / tan **DD**) (cos **SS** / cos **R1**)
= tan **R1** cos **SS** / tan **DD**
tan **P2** = cos **SS** / tan **DD**, and substitution yields:
sin **C5** = tan **R1** tan **P2**

tan **R1** = tan **SS** sin **DD**, and tan **P2** = cos **SS** / tan **DD**
Therefore, sin **C5** = (tan **SS** sin **DD**) (cos **SS** / tan **DD)**
= sin **SS** cos **DD**, an equation for **C5** in terms of **SS** and **DD**, independent of angle **R1**.

EQUATION for SAW BEVELS:

Extracting a kernel from "the stick"



Kernel extracted

DEFINITIONS of ANGLES:

Miter Line and **Angle**: The line or angle along which the saw travels.
Bevel Line and **Angle**: The angle on the adjacent face of the member.
Blade Angles: The saw blade angle setting as read on the gauge; normally, a reading of zero is at 90 degrees to the saw table.

We can now make the following identifications:

SS ⟶ 90 – **Blade Angle**
DD ⟶ **Miter Angle**
R1 ⟶ **Bevel Angle**
90 – **P2** ⟶ **Angle** on **Compound Face**
C5 ⟶ **Blade Angle** along **Adjacent Face**

<u>EQUATION for SAW BEVELS:</u>

Cognate Angles

Except for the actual values of the angles, the kernel extracted from the stick is in every way identical to the kernel of roof angles. All right angles are in the same locations. As for the other angles, what they are *named* is irrelevant, the *relationships* between angles remain the same.

Since tan **R1** = tan **SS** sin **DD**

tan **Bevel** = tan (90 – **Blade Angle**) sin **Miter**

Re-arranging the terms in the equation:

tan (**Blade Angle**) = sin **Miter** / tan **Bevel**

Consider the equation for **C5**:

tan **C5** = sin **R1** / tan **DD**

Substituting for angles in the same positions:

tan (**Adj. Blade Angle**) = sin **Bevel** / tan **Miter**

Note that in both cases, the tangent of the saw blade angle is the sine of the angle along which the saw is travelling, divided by the tangent of the angle on the adjacent face with respect to the proposed cut.

Observe that the angle on the face created by the cut occupies the same position as 90 – **P2** on the kernel of roof plane angles. Again, the relationships between the angles on both kernels remain the same, only the names have changed.

sin **P2** = cos **DD** cos **R1**

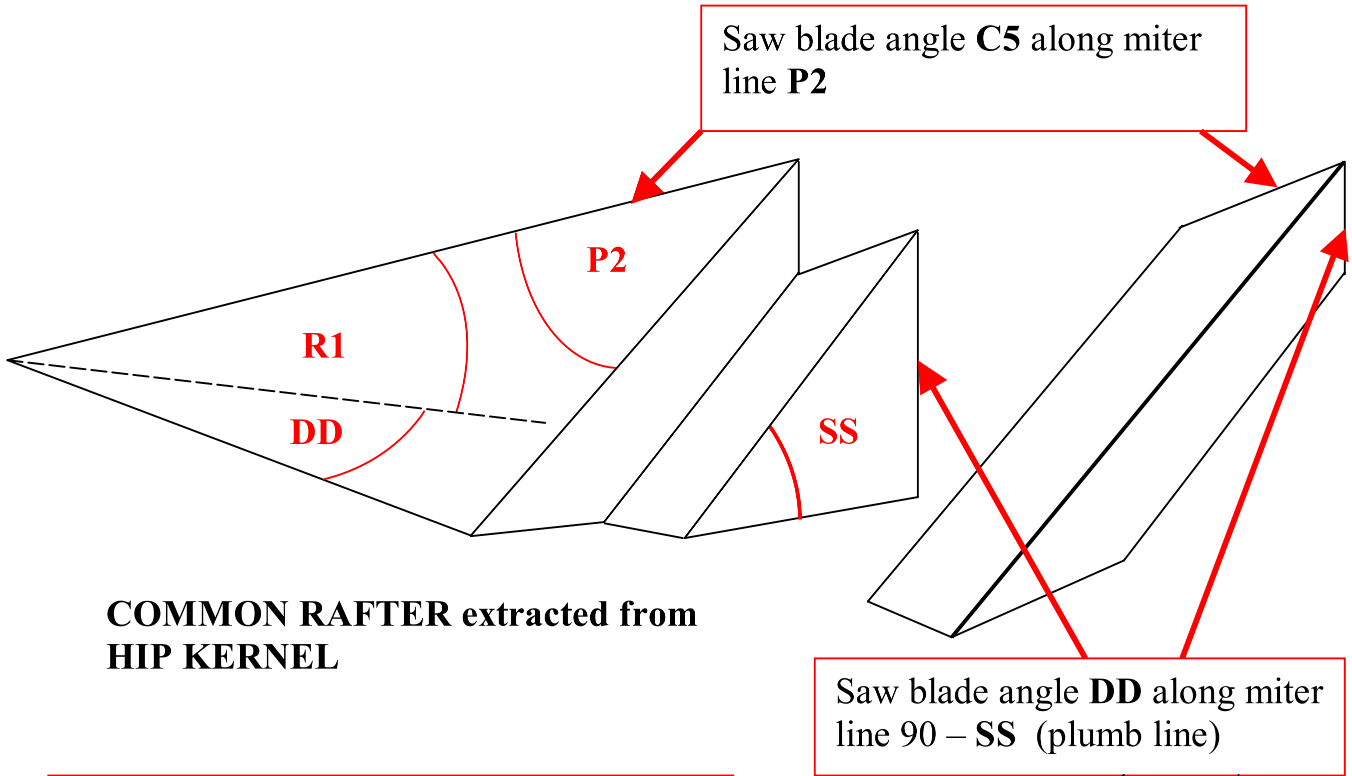
Therefore, cos (90 – **P2**) = cos **DD** cos **R1**,

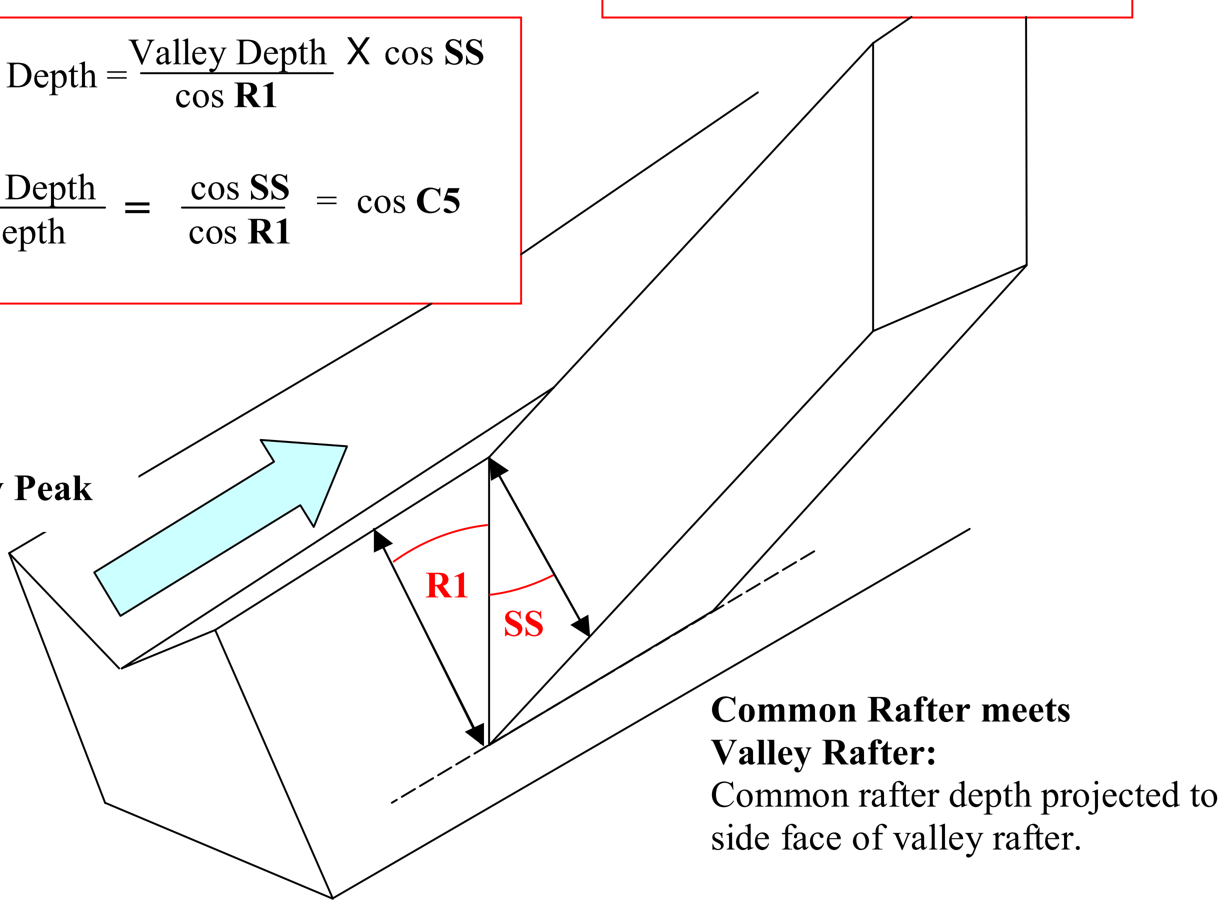
and cos (**Compound Face Angle**) = cos **Miter** cos **Bevel**

This formula is easy to remember, since it involves only the cosines of the angles concerned.

COMMON RAFTER ANGLES:

Common Rafter to Valley Rafter Depth Ratio:





$$\text{Common Depth} = \frac{\text{Valley Depth}}{\cos \mathbf{R1}} \times \cos \mathbf{SS}$$

$$\frac{\text{Common Depth}}{\text{Valley Depth}} = \frac{\cos \mathbf{SS}}{\cos \mathbf{R1}} = \cos \mathbf{C5}$$