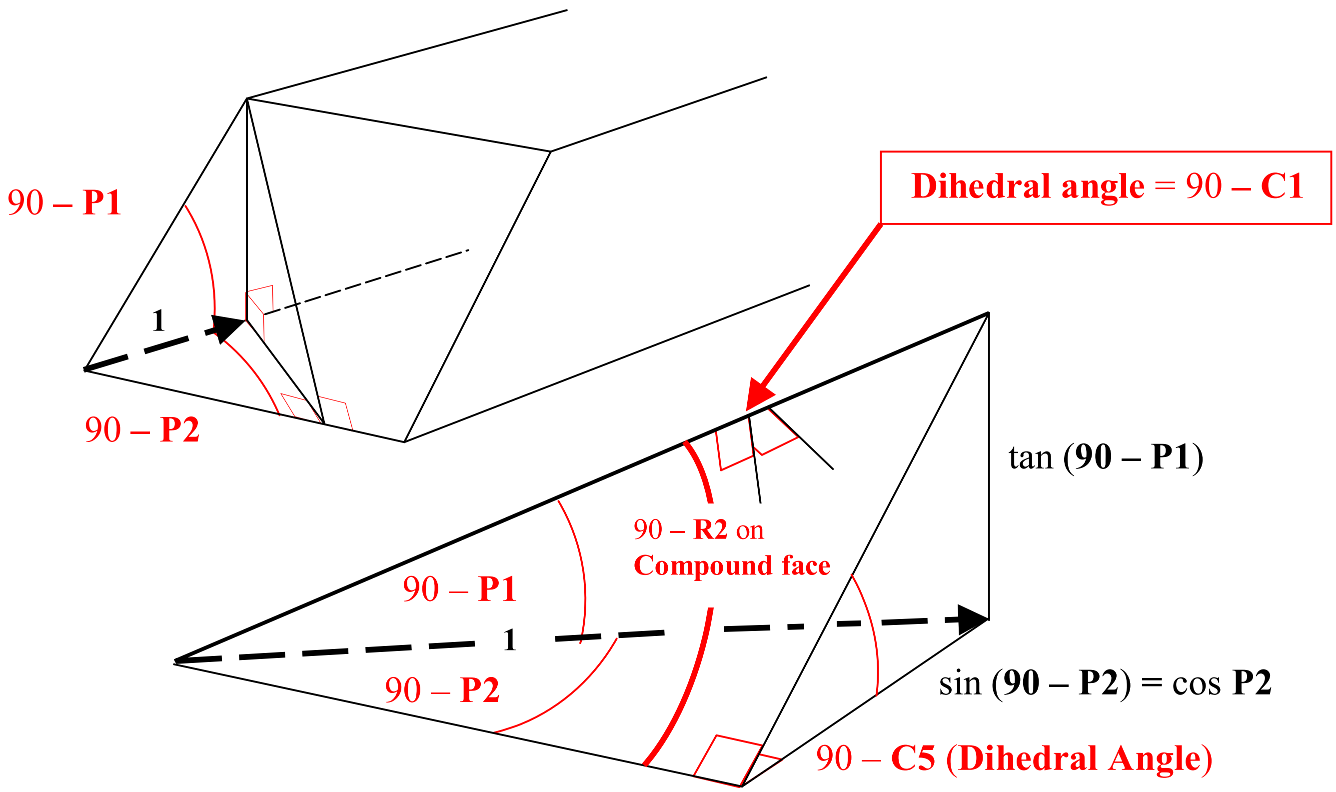
<u>PURLIN RELATED ANGLES</u>

Extracting a kernel from "the stick"



Kernel extracted from purlin

Make the following identifications:

SS ⟶ 90 – **C5**
DD ⟶ 90 – **P2**
R1 ⟶ 90 – **P1**
90 – **P2** ⟶ 90 – **R2**
C5 ⟶ **C1**

The values of two of the angles are known; 90 – **C5** is the dihedral angle between the face lying on the roof plane and the face which meets the side of the hip rafter, and 90 – **P2** is the required miter angle on the roof plane. The remaining angles may now be resolved.

PURLIN ANGLE RELATIONSHIPS:

C5, C1, P2, P1, R2

The equations relating sets of angles remain the same in any general kernel:

Since tan **R1** = tan **SS** sin **DD**

tan (90 – **P1**) = tan (90 – **C5**) sin (90 – **P2**)

Re-arranging the terms in the equation, and changing the trig functions of the complementary angles:

tan **P1** = tan **C5** / cos **P2**

Recall the equation for **C5**:

tan **C5** = sin **R1** / tan **DD**

Substituting for angles in the same positions:

tan **C1** = sin (90 – **P1**) / tan (90 – **P2**)

= tan **P2** cos **P1**

cos (90 – **P2**) = cos **DD** cos **R1**

Therefore, cos (90 – **R2**) = cos (90 – **P2**) cos (90 - **P1**),

and sin **R2** = sin **P2** sin **P1**

These are not the only possible equations. Going back to the kernel of primary roof angles, note that it was possible to obtain formulas for given angles in terms of the sine, cosine, or tangent. Say we prefer the equation for **R2** to be expressed in terms of the tangent of the angle, more suitable for layout with a framing square.

Since tan **P2** = cos **SS** / tan **DD**, we may substitute for angles in the same positions, resulting in:

tan **R2** = cos (90 – **C5**) / tan (90 – **P2**)

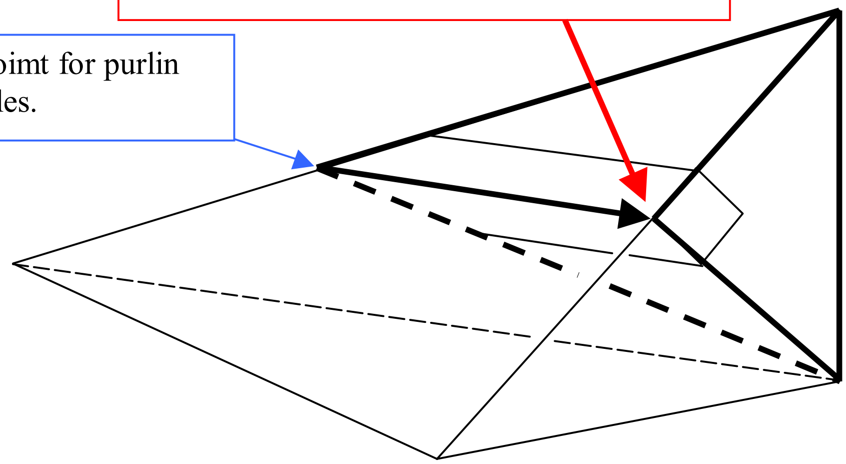
Re-arranging terms and changing the trig functions:

tan **R2** = tan **P2** sin **C5**, an equation in terms of the original roof angles.

EXTRACTING the PURLIN KERNEL from the HIP KERNEL:

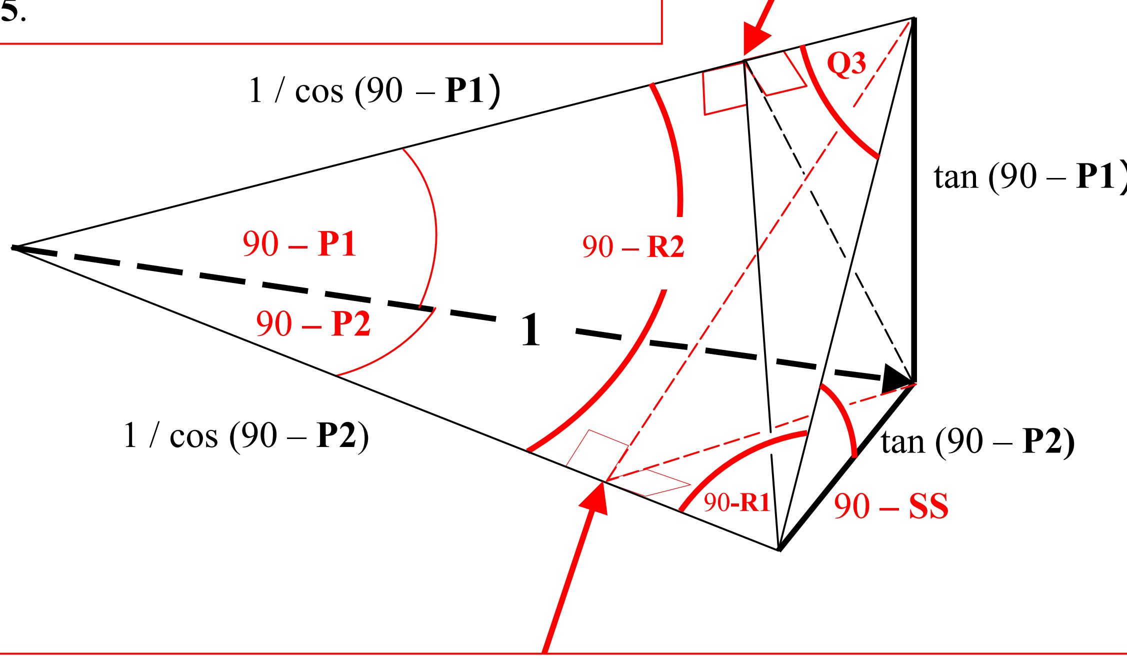
Purlin kernel, as situated on general Hip kernel. Note the location of the three mutually perpendicular lines.

Working poimt for purlin related angles.



Purlin kernel extracted and rotated, with the angles labeled. The kernel geometry is the same as that of the kernel used to resolve angles **P2** and **C5**.

Dihedral Angle =
90 – **C1**



The kernel may be divided along the lines of dihedral angle 90 – **C5**, resulting in two kernels: the kernel of Common rafter angles, and a kernel the same as that extracted from the stick. However, this kernel will be analyzed as it sits, to demonstrate that there is more than one method, or one type of geometry, whereby it is possible to obtain angle formulas.

EXTRACTING the PURLIN KERNEL from the HIP KERNEL:

The following angles are cognates, that is, they occupy the same positions in their respective kernels:

90 – **SS** ⟶ **A7**, the plumb backing angle, which has not been solved at this point, but the diagram clearly shows that:

tan (90 – **SS**) = tan (90 – **P1**) / tan (90 – **P2**)
Therefore, tan **P1** = tan **SS** tan **P2**

90 – **P2** ⟶ **DD**
90 – **P1** ⟶ **R1**
90 – **C1** ⟶ 90 - **C5**
90 – **R2** ⟶ 90 – **P2**
90 – **C5** ⟶ **SS**

Since cos **C5** = cos **SS** / cos **R1**
cos **C1** = cos (90 – **C5**) / cos (90 – **P1**)
= sin **C5** / sin **P1**

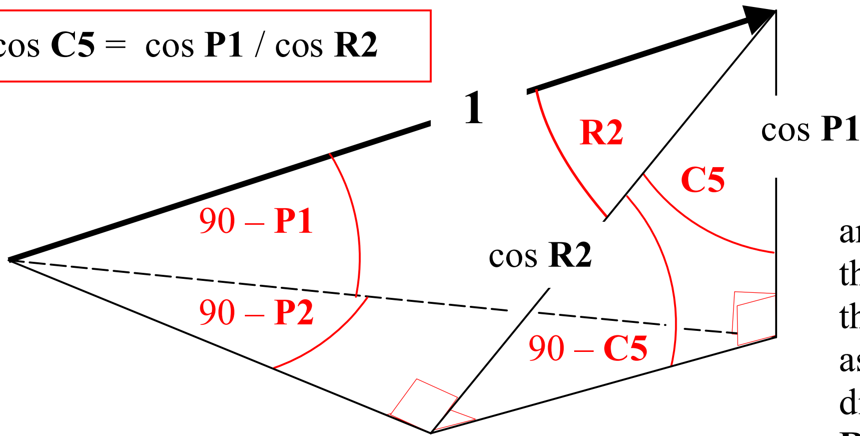
Also, if cos **P2** = sin **DD** / cos **C5**
then cos **R2** = sin (90 – **P2**) / cos **C1**
= cos **P2** / cos **C1**

Other sets of equations, and different relationships between angles, may be generated by dividing the Main kernel into two standard Hip kernels along the lines of dihedral angle 90 – **C1**, and defining angle **Q3** = **R1** + **R2**.

CHANGING the REFERENCE LENGTH (Unit vector):

Purlin Depth to Valley Rafter Depth Ratio:

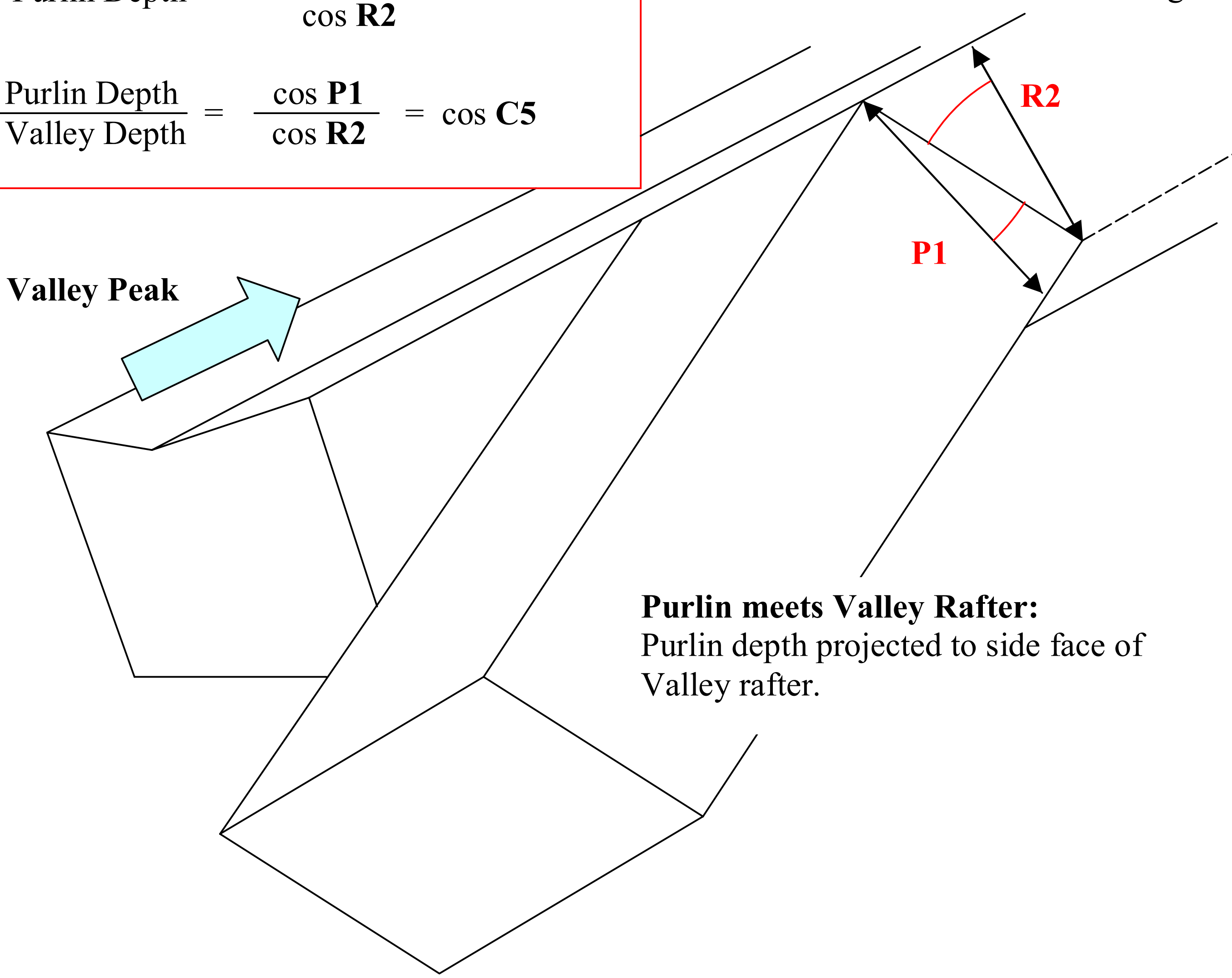
$$\cos \mathbf{C5} = \cos \mathbf{P1} / \cos \mathbf{R2}$$



Compare this kernel, and the original diagram of the Purlin kernel situated on the Hip kernel. The line assigned the unit value in this diagram defined angle 90 – **R2** in the original diagram (dashed line). In the sketch below, either of the lines may be used as a reference length.

$$\text{Purlin Depth} = \frac{\text{Valley Depth}}{\cos \mathbf{R2}} \times \cos \mathbf{P1}$$

$$\frac{\text{Purlin Depth}}{\text{Valley Depth}} = \frac{\cos \mathbf{P1}}{\cos \mathbf{R2}} = \cos \mathbf{C5}$$



Purlin meets Valley Rafter:
Purlin depth projected to side face of Valley rafter.