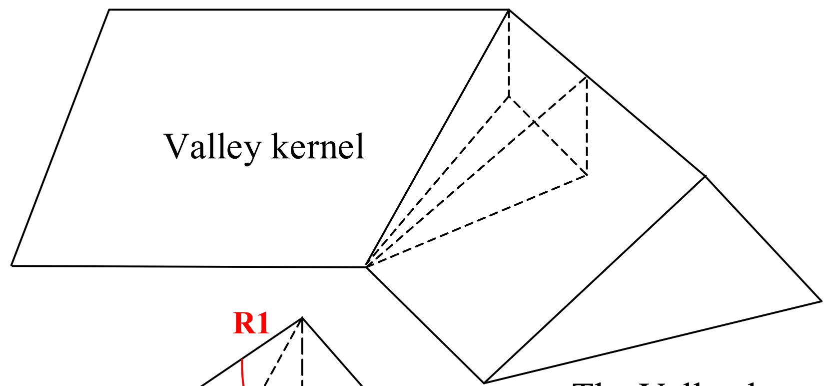
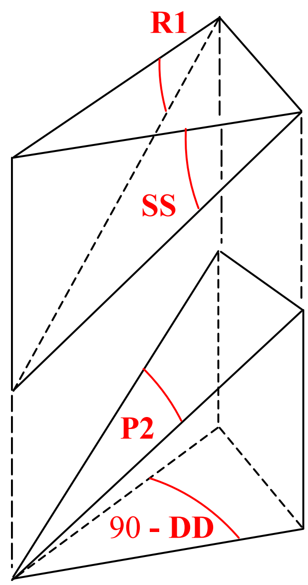
HIP / VALLEY KERNELS

Dihedral angle turned through an angle of revolution



Valley kernel extracted, and compared to Hip kernel.

The deck is defined as a level plane passing through the the intercept of the ridges.



The Valley kernel may be interpreted as an inverted Hip kernel. Note the locations of angles **DD**, and 90 – **P2**. Normally situated at the base or foot of the hip rafter, they are now at the valley peak.

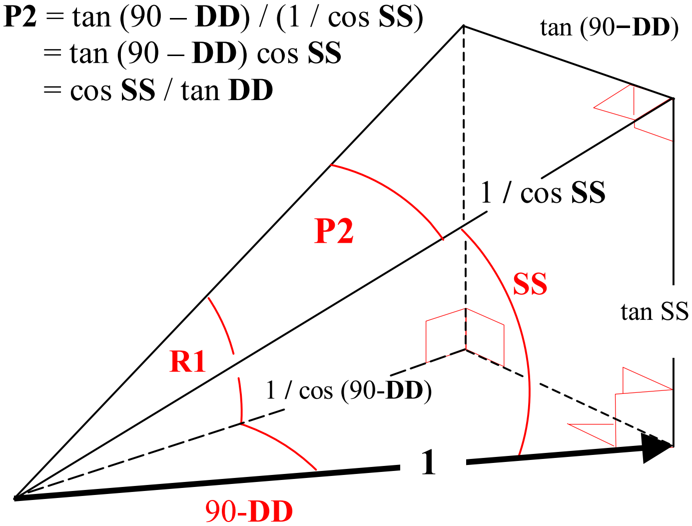
Valley related angles **R4**, **R5**, and **A5**, which will be analyzed later, also behave in this manner.

Let the radius vector (common run) = **1** :

$$\tan \mathbf{R1} = \tan \mathbf{SS} \,/\, (1 / \cos(90 - \mathbf{DD}))$$
$$= \tan \mathbf{SS} \cos(90 - \mathbf{DD})$$
$$= \tan \mathbf{SS} \sin \mathbf{DD}$$

Similarly:

$$\tan \mathbf{P2} = \tan(90 - \mathbf{DD}) \,/\, (1 / \cos \mathbf{SS})$$
$$= \tan(90 - \mathbf{DD}) \cos \mathbf{SS}$$
$$= \cos \mathbf{SS} / \tan \mathbf{DD}$$



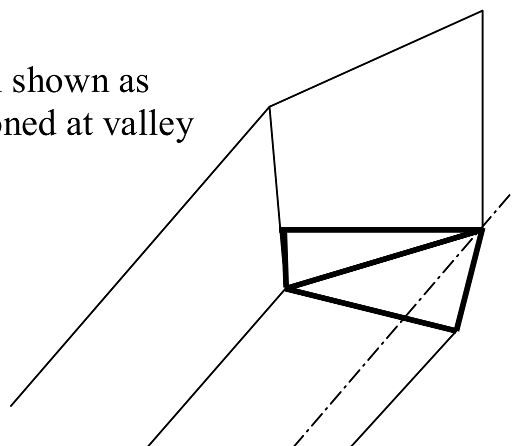
Observe that we have a general equation for the revolution of any angle, given this scenario. The tangent of the resulting angle on the cutting plane (**R1**, or **P2**, as the case may be) equals the tangent of the original dihedral angle multiplied by (the same as division by the reciprocal) the cosine of the angle of revolution.

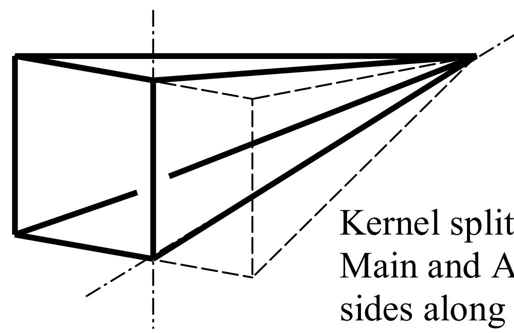
VALLEY ANGLE RELATIONSHIPS:

R4B, R5B, A5B, DD, R1

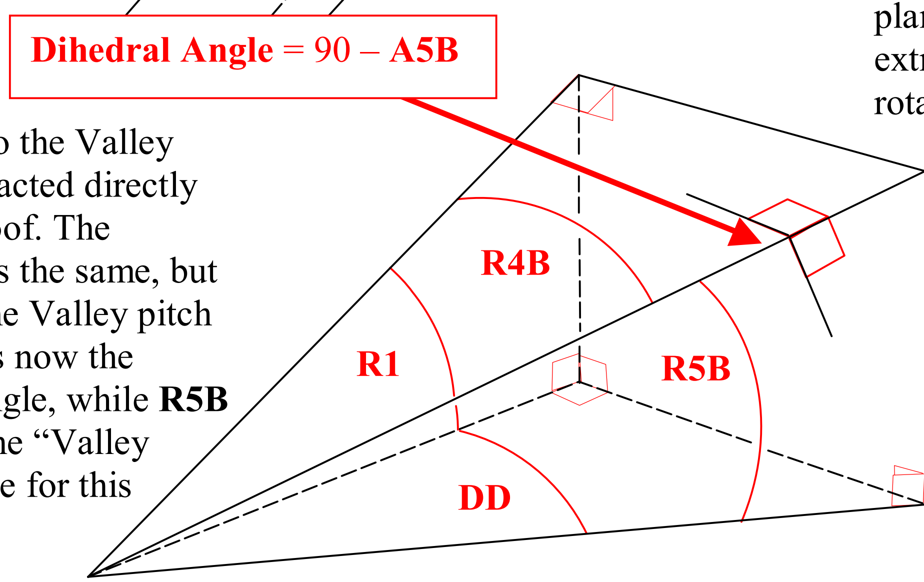
Extracting a "Bird's-Mouth" Kernel at the Valley Peak

Kernel shown as positioned at valley peak.





Kernel split into Main and Adjacent sides along the plane of angle **R1**, extracted, and rotated.



Compare to the Valley kernel extracted directly from the roof. The geometry is the same, but note that the Valley pitch angle **R1** is now the dihedral angle, while **R5B** serves as the "Valley pitch" angle for this model.

Kernel with the angles labeled. Remember that the standard model is a Hip kernel (refer to the diagram on the previous page). The angles located at the foot, or ***base***, of a hip rafter will be found at the ***peak*** of a valley rafter.

For future reference, also note the location of dihedral angle 90 – **A5B**, which governs the value of the saw blade setting. Like angle **C5** on the actual roof, **A5B** lies along the line between the "roof plane" of the kernel and the plumb plane through the long axis of the "hip" or "valley" on the kernel. Alternatively, consider the angle **R4B** as the miter angle, and 90 **– R1** as the bevel angle.

VALLEY RAFTER ANGLE FORMULAS:

R4B, **R5B**, **A5B**, **DD**, **R1**

At this point, we have formulas for the rotation of one angle through another angle.

Angles **R1** and **DD** are known quantities. Given this information, we can calculate **R4B**, the miter angle on the bottom face or shoulder of the valley rafter; simply rotate angle **DD** through angle **R1**:

tan **R4B** = tan **DD** cos **R1**

Angle **R5B** is the complement of the angle on the plane created by cutting a compound angle. We can find the value of **R5B** by rotating angle **R1** through angle **DD**:

tan **R5B** = tan **R1** cos **DD**

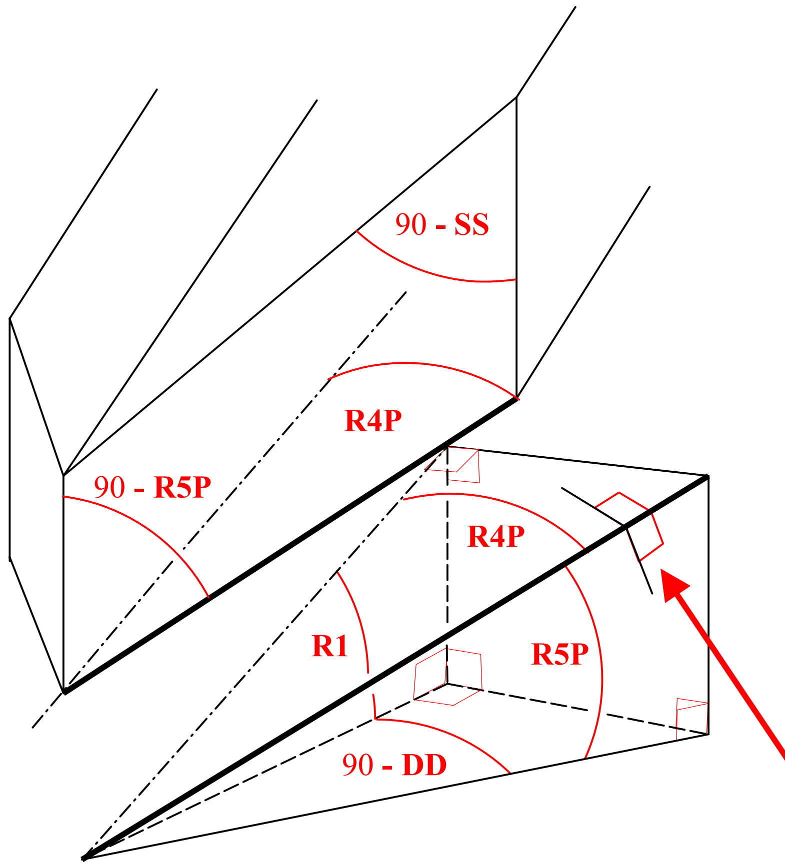
Another solution:
If cos (**Compound Face Angle**) = cos **Miter** cos **Bevel**
then cos (90 − **R5B**) = cos **R4B** cos (90 − **R1**)
and sin **R5B** = cos **R4B** sin **R1**

As for angle **A5B**:
Since tan (**Blade Angle**) = sin **Miter** / tan **Bevel**
tan **A5B** = sin **R4B** / tan (90 − **R1**)
= sin **R4B** tan **R1**

By comparing the angles in the "Bird's-mouth" kernel to the Valley and Hip kernels, and making appropriate substitutions, it is possible to find further relationships. However, for the time being, instead of dealing with abstract models that may be difficult to relate to the real world, the focus will be on the simplest calculations and geometry that may be derived from an examination of the proposed cut.

<u>VALLEY RAFTER ANGLE FORMULAS</u>:

R4P, R5P, A5P, 90 - DD, R1



To calculate angles at the base or foot of a valley rafter, consider a kernel positioned below the rafter as shown in the diagram to the left. The “roof plane” of the kernel is parallel to the bottom shoulder of the valley rafter (imagine the rafter seated on the kernel).

Since a valley rafter is in essence an “upside-down” hip rafter, the angular values expected at the ***peak*** of a hip rafter are located at the ***base*** of a valley rafter.

Using the angle rotation formulas:

tan **R4P** = tan (90 – **DD**) cos **R1**
= cos **R1** / tan **DD**

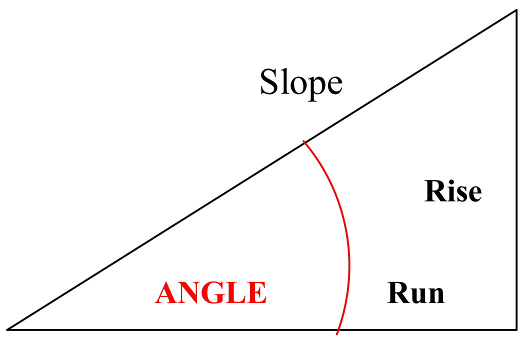
tan **R5P** = tan **R1** cos (90 – **DD**)
= tan **R1** sin **DD**

To determine the value of **A5P**, substitute the appropriate quantities in the equation for the saw blade angle:

If tan (**Blade Angle**) = sin **Miter** / tan **Bevel**
then tan **A5P** = sin **R4P** / tan (90 – **R1**)
= sin **R4P** tan **R1**

<u>Notes re: Angle Formulas</u>

When working with a framing square, the calculations for miter, bevel and cutting angles are best if given in terms of the tangent of the required angle. Angles are expressed as a value "over-12", and since the tangent = rise / run, we have a trig function of a required angle suited for direct use on the square.



Generally:
Rise = Run X tan (**ANGLE**)

For "over-12" measurements:
Rise = 12 X tan (**ANGLE**)

If using a programmable calculator or spreadsheet to determine angular values, the tangent of an angle is not necessarily the best mode of calculation, since trig functions change sign according to quadrant. Recall that given a Total Deck Angle > 90 degrees, it is possible for either **DD** or **D** to exceed 90 degrees. Subsequent calculations will be affected by the trig function chosen; the cosine of the angle always returns a positive value for the angles listed below.

The formulas were resolved using linear algebra, and are given without proof. Relationships between the peak and base values may be supplementary, rather than complementary, depending on the value of **DD** (base or peak) entered. Dihedral angle related values **C5** and **A5** may be 90 plus ***or*** minus the angle.

$\cos(90 \pm \mathbf{C5}) = \sin \mathbf{SS} \cos \mathbf{DD}$

$\cos \mathbf{R1} = \cos \mathbf{SS} / \sin(90 \pm \mathbf{C5})$

$\cos \mathbf{P2B} = \cos \mathbf{DD} \cos \mathbf{R1}$

$\cos(90 \pm \mathbf{A5B}) = \sin \mathbf{R1} \sin \mathbf{DD}$

$\cos \mathbf{R5B} = \cos \mathbf{R1} / \sin(90 \pm \mathbf{A5B})$

$\cos \mathbf{R4B} = \cos \mathbf{DD} / \sin(90 \pm \mathbf{A5B})$