RAFTER and PURLIN FORMULAS:

It would be an exercise in futility to attempt to enumerate *all* the possible formulas, as every case is different. Knowledge of how the angles relate to the overall roof structure and to each other are the keys to successful results. First, some formulas for working on the roof plane:

JACK RAFTERS: Given spacing O.C. on Eave / Ridge

Difference in Lengths = (Spacing O. C.) X tan (90 – **P2**)

= (Spacing O. C.) ÷ tan **P2**

Spacing on Valley = (Spacing O. C.) ÷ sin **P2**

= (Spacing O. C.) ÷ cos (90 – **P2**)

Jack Rafter Depth = (Valley Depth X cos **SS**) ÷ cos **R1** **

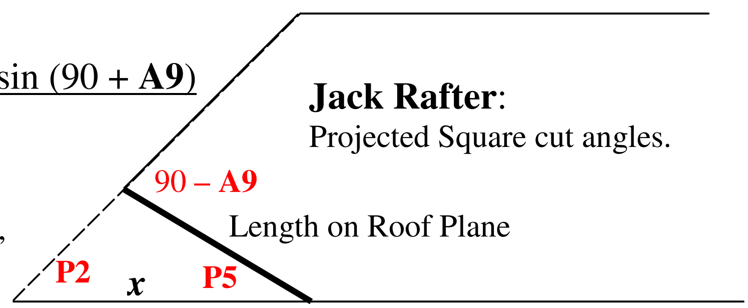
Housings: Length on Roof Plane = Housing Depth ÷ cos **A7**

> ** Note: $\frac{\cos \mathbf{SS}}{\cos \mathbf{R1}} = \cos \mathbf{C5}$

x = Distance from Rafter end

$$= \frac{^{*}\ (\text{Length on Roof Plane}) \times \sin(90 + \mathbf{A9})}{\sin \mathbf{P2}}$$

* Law of Sines: A similar relation can be used in conjunction with **P5**, and angle **P4** on the Jack Purlin.



JACK PURLINS: Given spacing O.C. on Common Rafter

Difference in Lengths = (Spacing O. C.) X tan **P2**

= (Spacing O. C.) ÷ tan (90 – **P2**)

Spacing on Valley = (Spacing O. C.) ÷ cos **P2**

= (Spacing O. C.) ÷ sin (90 – **P2**)

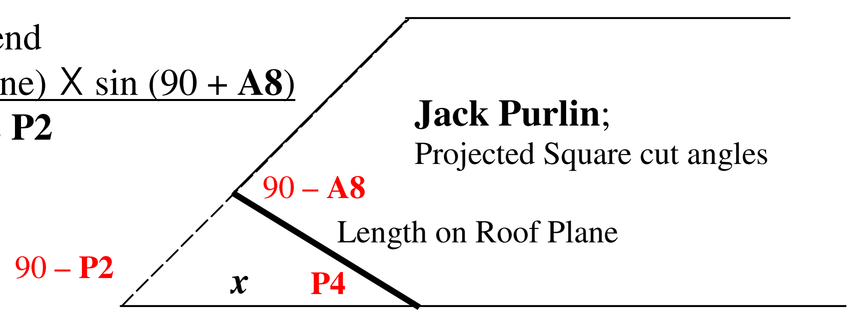
Jack Purlin Depth = (Valley Depth X cos **P1**) ÷ cos **R2** **

Housings: Length on Roof Plane = Housing Depth ÷ sin **Q1**

> ** Note: $\frac{\cos \mathbf{P1}}{\cos \mathbf{R2}} = \cos \mathbf{C5}$

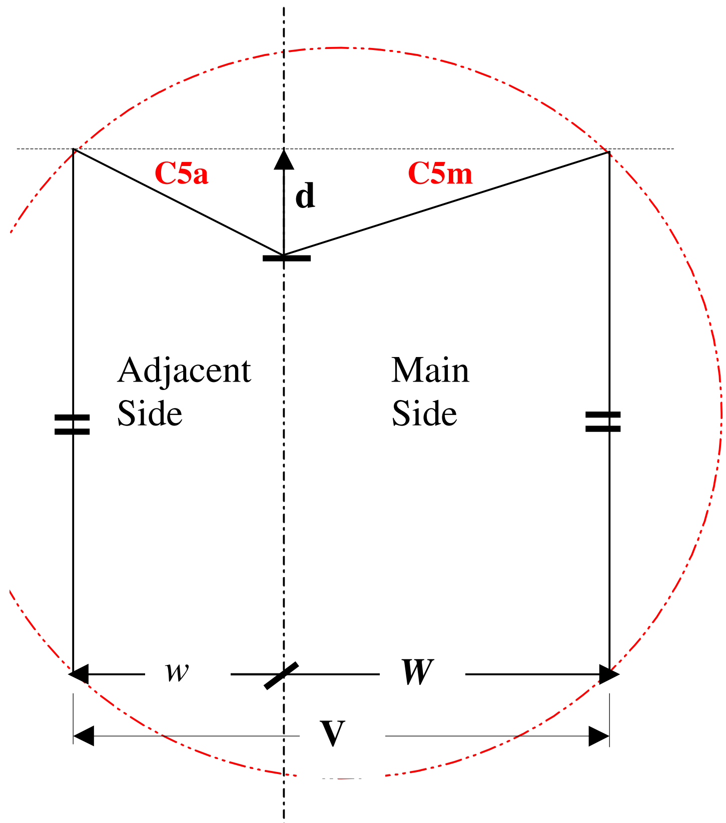
x = Distance from Purlin end

$$= \frac{(\text{Length on Roof Plane}) \times \sin(90 + \mathbf{A8})}{\cos \mathbf{P2}}$$



OFFSETTING the VALLEY TROUGH LINE:

In the event that the pitches and deck angles are irregular, this layout will force the transverse axis (width) of the Valley to be at right angles to the length. Subsequent joinery is simplified, and as an aesthetic consideration, equal arcs will be visible on each side of the log Valley rafter cutting.



Overall **VALLEY WIDTH** = **V** = ***W*** + *w*

$\boldsymbol{W} = \mathbf{V} \tan \mathbf{DD} \div (\tan \mathbf{DD} + \tan \mathbf{D})$

$w = \mathbf{V} \tan \mathbf{D} \div (\tan \mathbf{DD} + \tan \mathbf{D})$

Consider two Valley peaks that meet at a center post. A similar calculation with the **R4B** angles can be carried out directly on the Valley rafter.

$\boldsymbol{W} = \mathbf{V} \tan \mathbf{R4Bm} \div (\tan \mathbf{R4Bm} + \tan \mathbf{R4Ba})$

$w = \mathbf{V} \tan \mathbf{R4Ba} \div (\tan \mathbf{R4Bm} + \tan \mathbf{R4Ba})$

Both sets of calculations return the same respective values for ***W*** and *w*. Assume that the Main side pitch is the lesser value, hence **DD** is greater than **D**. Common sense dictates that ***W*** should be greater than *w*.

The figures may further be checked using the formula:

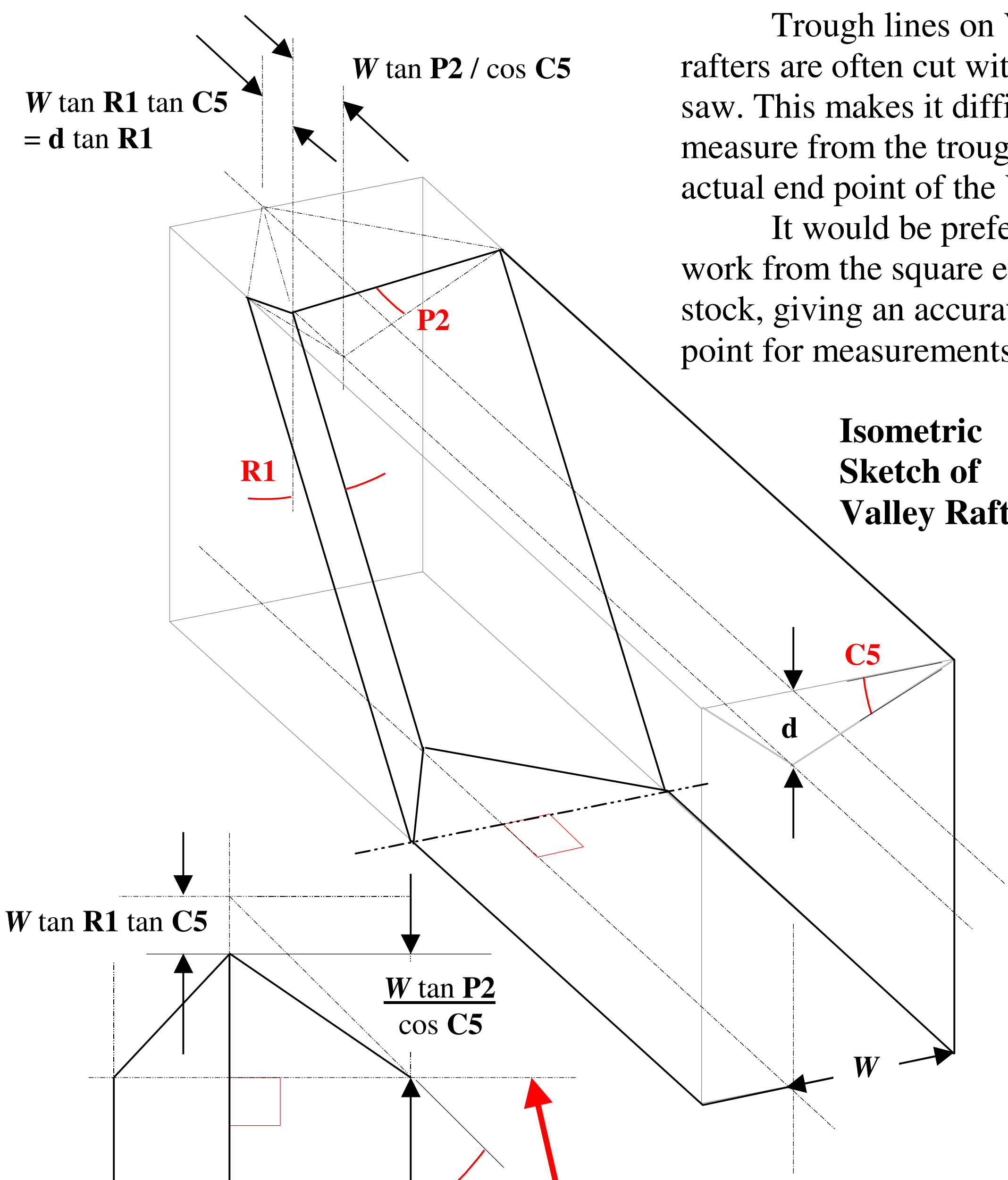
$$\mathbf{d} = \boldsymbol{W} \tan \mathbf{C5m} = w \tan \mathbf{C5a}$$

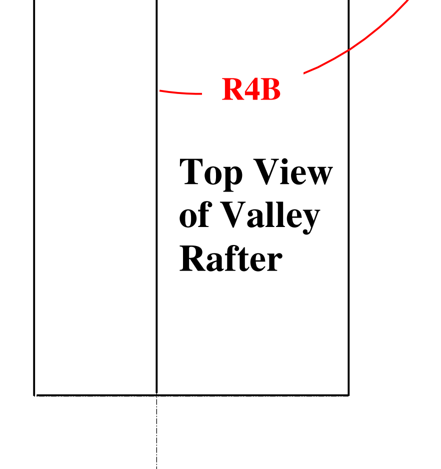
In other words, there can only be one resulting dimension for the trough line depth. **Note how the dimensions can always be double-checked, since they are linked by more than one set of angles**.

<u>VALLEY PEAK: MOVING the WORKING POINT</u>:

Trough lines on Valley rafters are often cut with a chain saw. This makes it difficult to measure from the trough or the actual end point of the Valley.

It would be preferable to work from the square end of the stock, giving an accurate reference point for measurements.





Working from the designated reference line

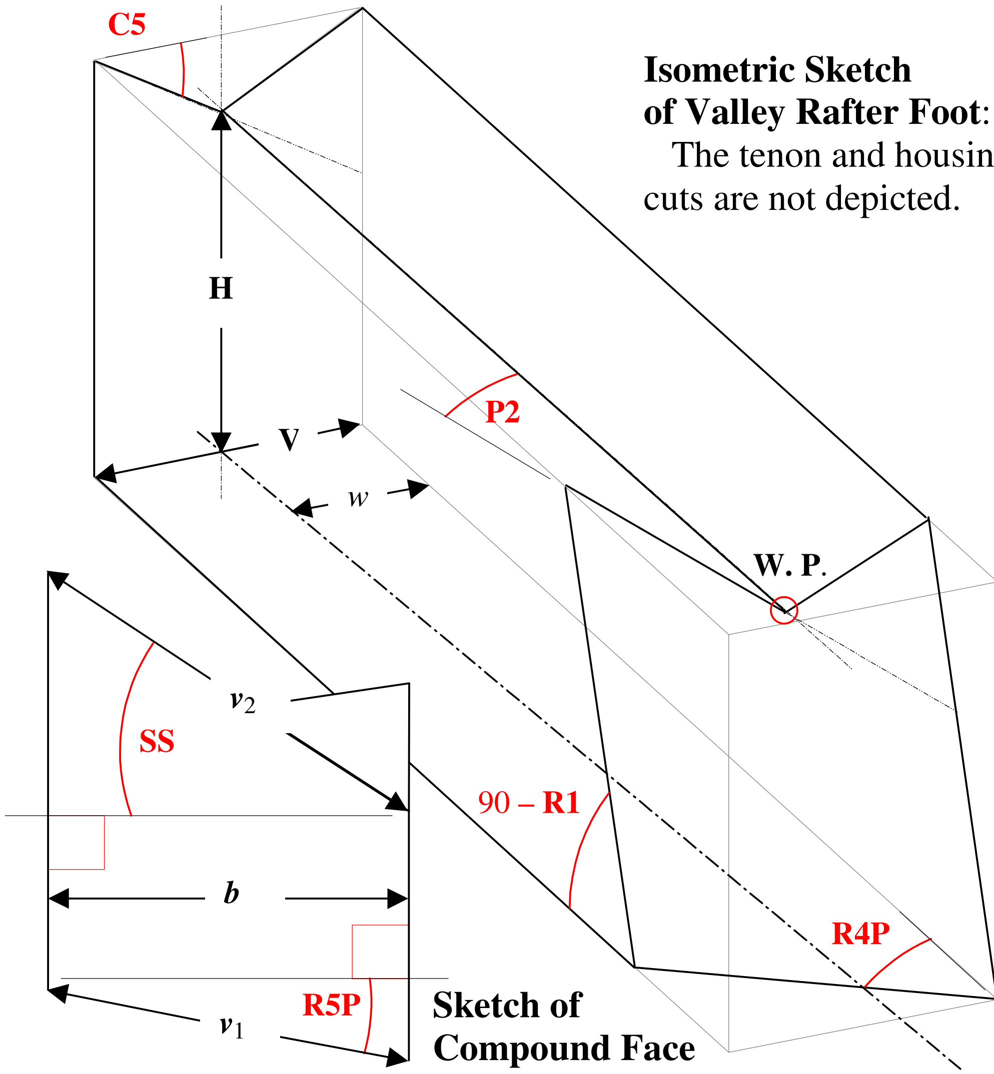
W tan **P2** / cos **C5** + *W* tan **R1** tan**C5**

= *W* (tan **P2** / cos **C5** + tan **R1** tan**C5**)

= *W* / tan **R4B**.

This is the anticipated angle at the Valley peak or ridge before the backing angle is cut. Again, note how more than one calculation may be used to check the dimensions.

VALLEY FOOT at COMMON RAFTER:



Let the overall Valley width = **V**:

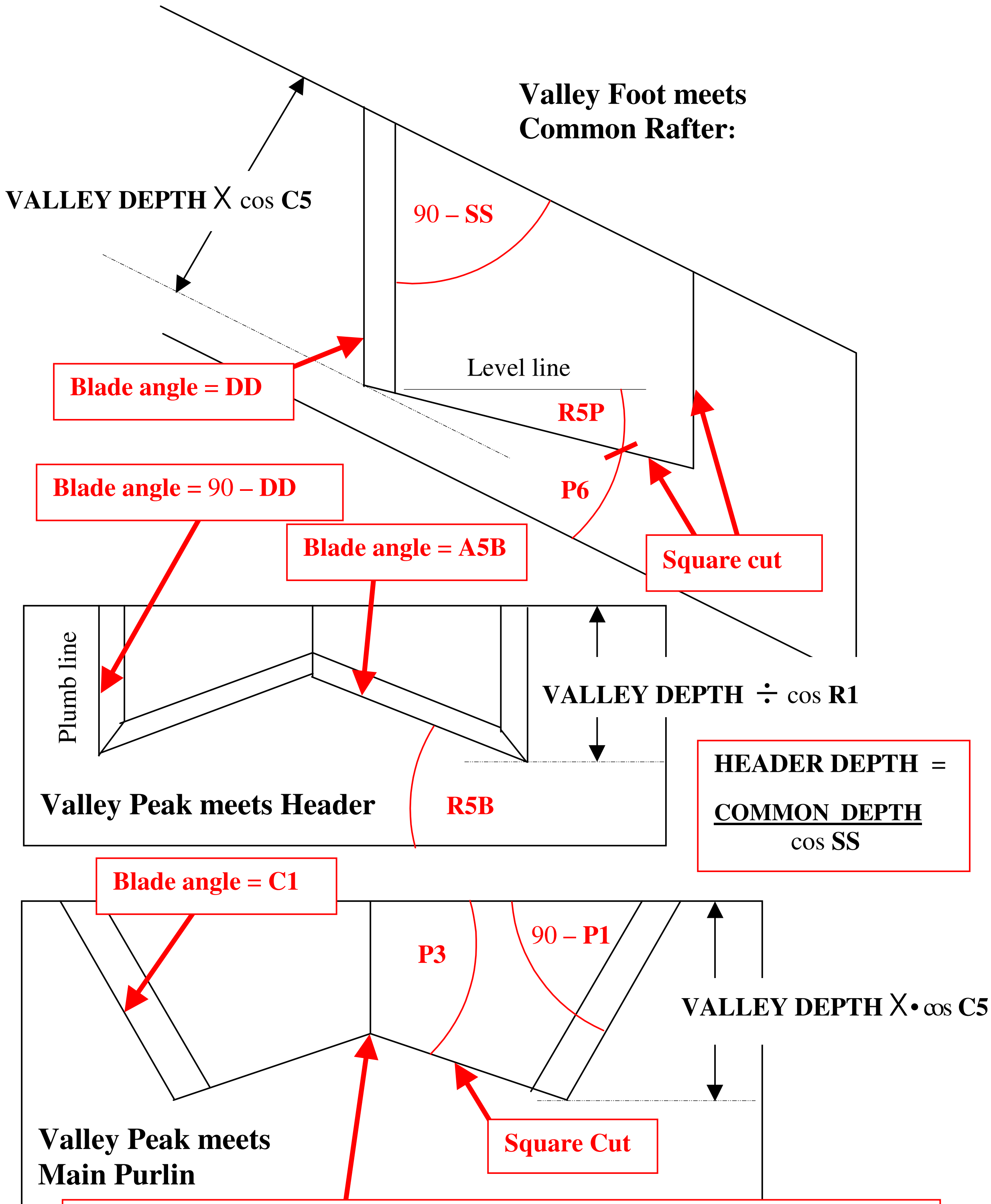
The dimension along the bottom face, v_1 , is **V** ÷ sin **R4P**
Projecting the same measurement through the upper face angles:

v_2 = **V** ÷ (cos **C5** sin **P2**) … … or, **V** ÷ sin **C1**
The width across the compound face is:
$b = v_2$ cos **SS** = v_1 cos **R5P**

Note that **V** cos **SS** ÷ sin **C1** = **V** cos **R5P** ÷ sin **R4P** = **V** ÷ cos **DD**
A sketch of the Valley rafter in plan will confirm this result.
Stock length: Given the measurement to the Working Point, add

(**H** tan **R1**) + (*w* ÷ tan **R4P**)

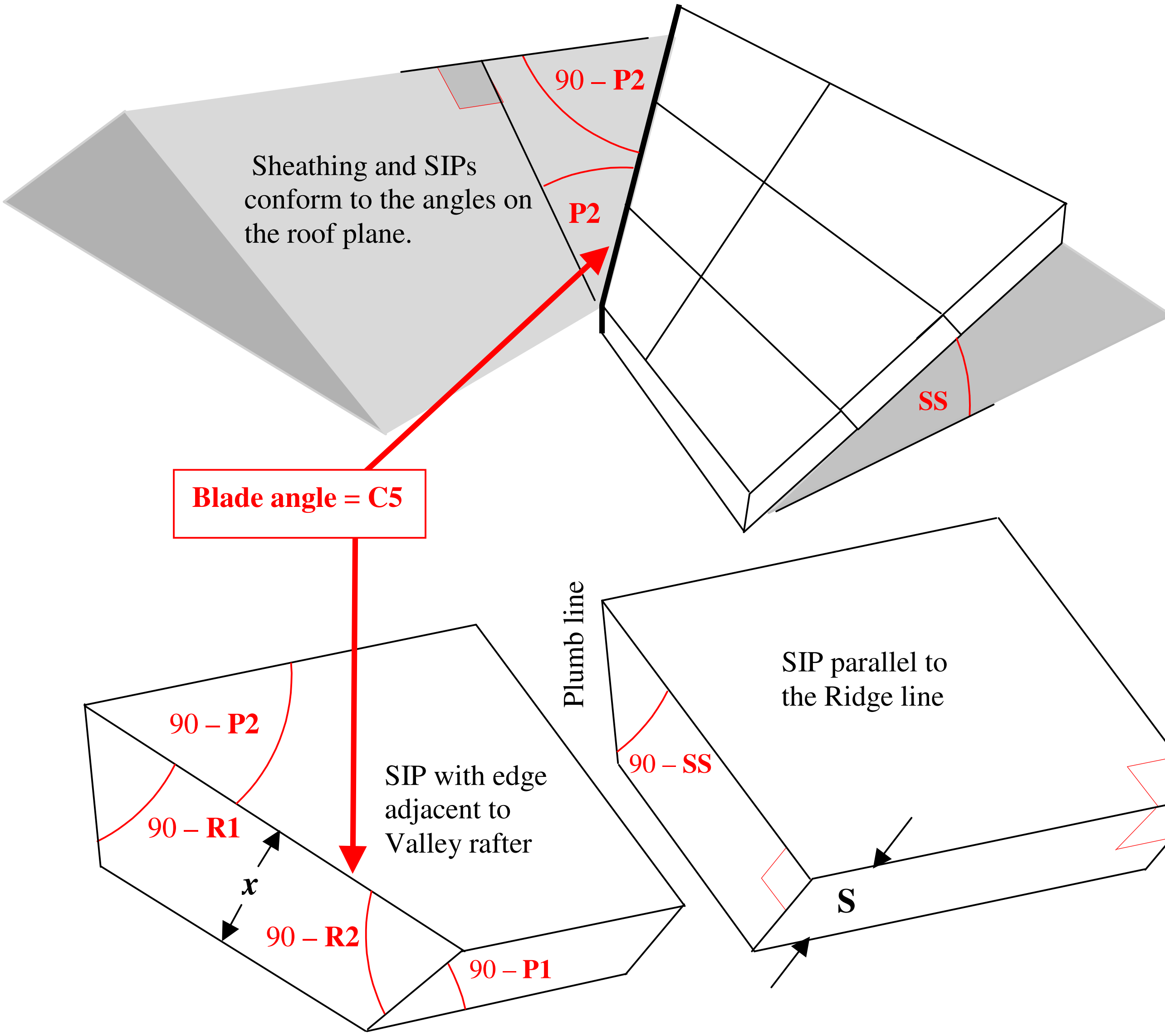
<u>MORE VALLEY RELATED RATIOS</u>:



Angle **Q4** is required as a “correction factor” at the bottom faces of the adjacent Valleys; the cut here is not a square cut. Note how the intercepting planes create same angles and lines as the **Purlin meets Valley** joint, but arranged differently.

DIMENSIONING SIPs and SHEATHING:

Complex roof system angles are not limited to dimensioning the logs and timbers. The angles govern all the materials used in the roof, including hardware such as gussets. Sheathing would simply follow the **P2** angles. The SIPs in the following example lie transverse to the rafters; they are in essence purlins and therefore are treated as such.



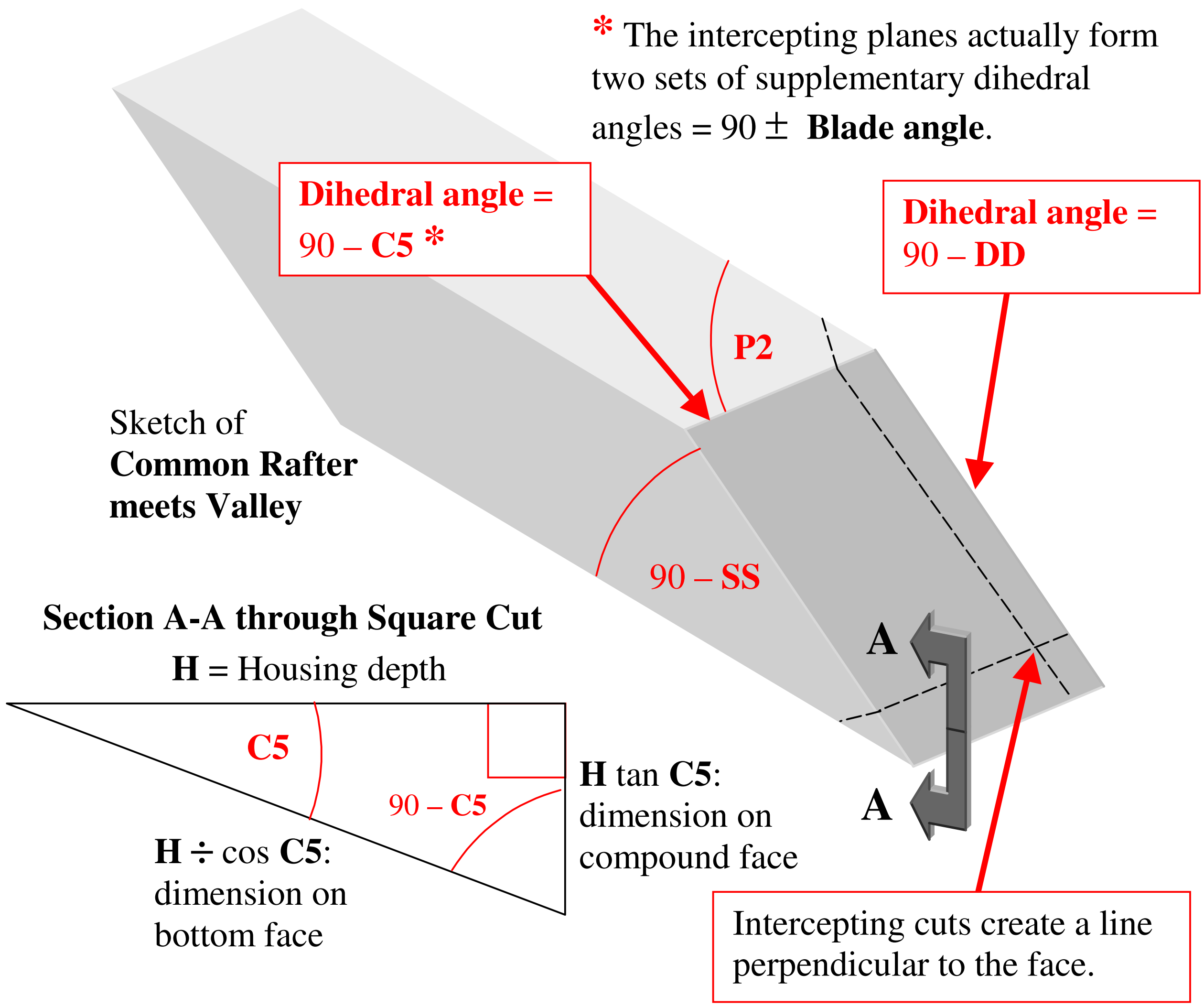
The plane of x follows the side face of the Valley, let the thickness of the SIP = **S**:

$x = (\mathbf{S} \cos \mathbf{R2}) \div \cos \mathbf{P1} = (\mathbf{S} \cos \mathbf{R1}) \div \cos \mathbf{SS} = \mathbf{S} \div \cos \mathbf{C5}$

The same ratios are to project the depth of a purlin (or, a common rafter) to the side face of the Valley.

The depth measured along the ridge line would be $\mathbf{S} \div \cos \mathbf{SS}$, the formula used to project the common rafter depth to a header.

<u>SQUARE CUTS: HOUSINGS and TENONS</u>:



The Square cuts, by definition perpendicular to the Compound face of the tenoned member, intercept to create a line perpendicular to the face. Defined as the **Housing depth**, this dimension serves as the reference length for dimensioning *both* of the Square cuts. Note that the compound face is equivalent to the face being mortised, in this case the Valley rafter side face.

A tenon is treated as an extension of the housing; mortise angles and measurements are equal to their corresponding tenon counterparts.

Angle **C5** was used in the above example, but the method may be applied to all such cuts by substituting the appropriate angles. The angles in question are always known: the **Blade angles** along the miter and bevel of the tenoned member.