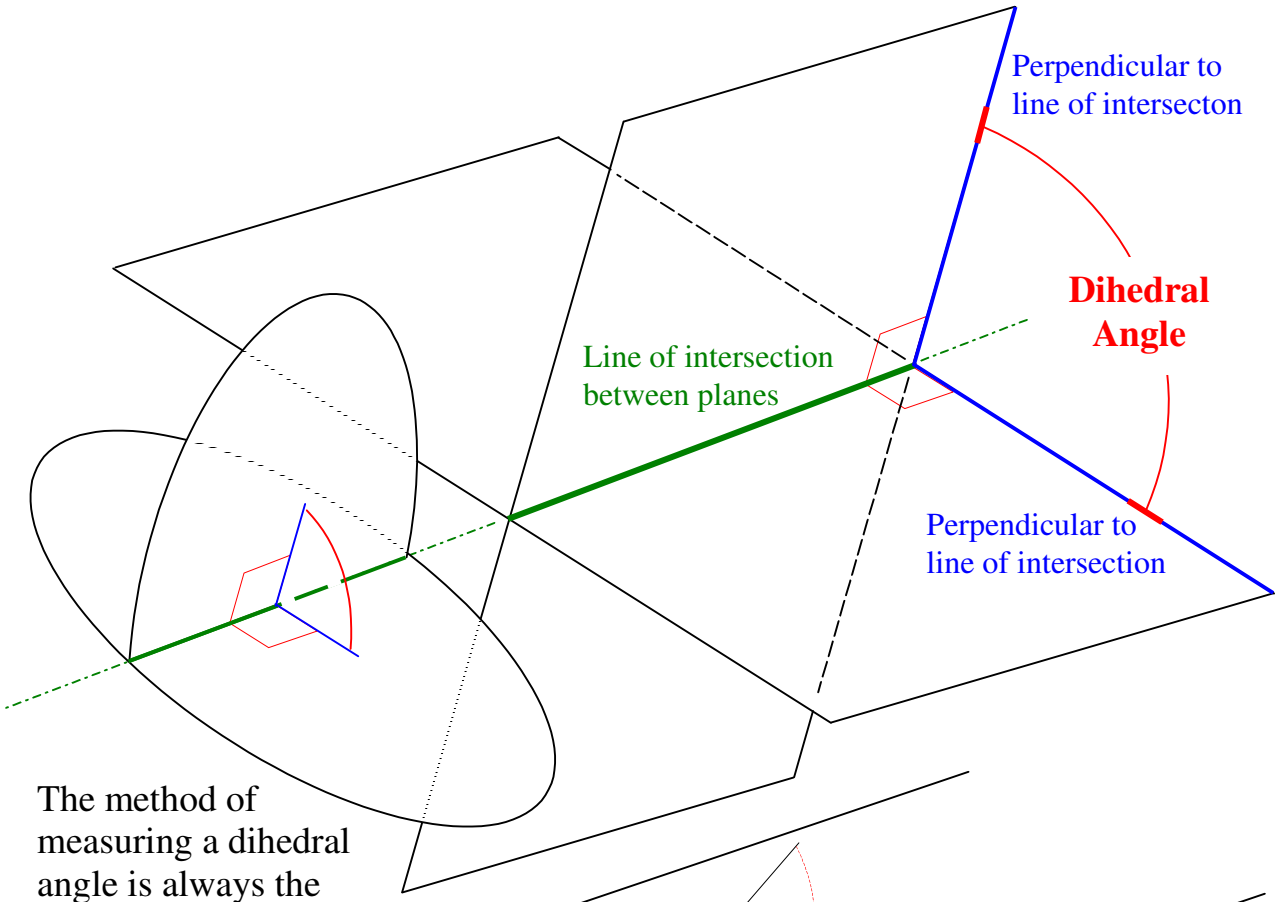
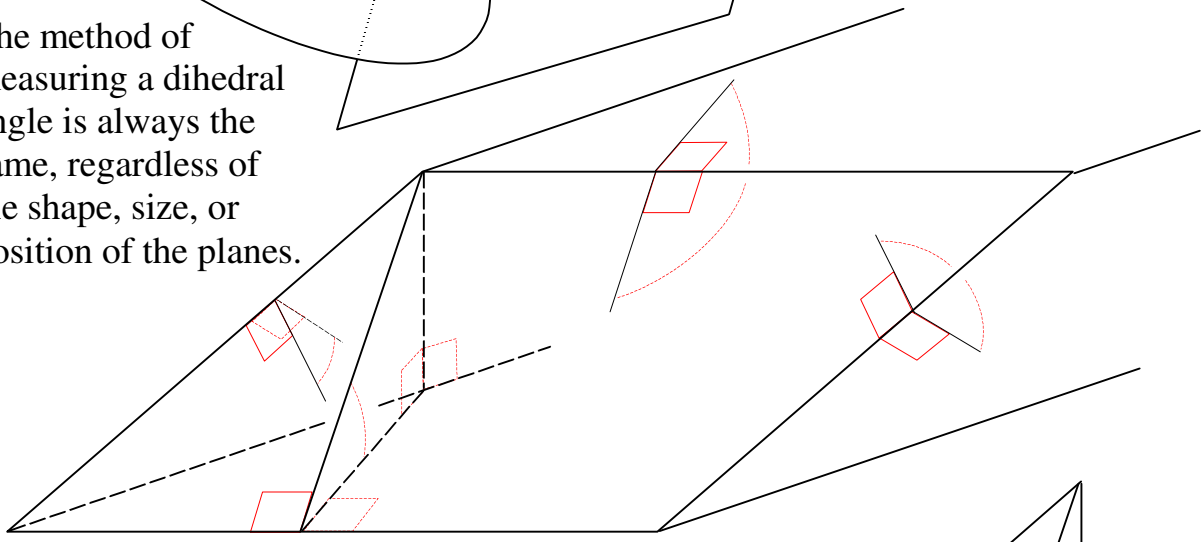
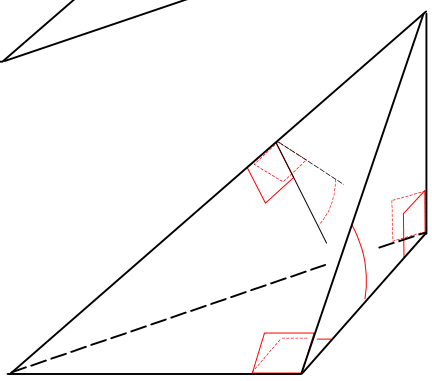
DIHEDRAL ANGLES

Definition: The angle between two planes, measured from perpendiculars to the line created by the intersection of the planes.



The method of measuring a dihedral angle is always the same, regardless of the shape, size, or position of the planes.





Typical Geometric Models

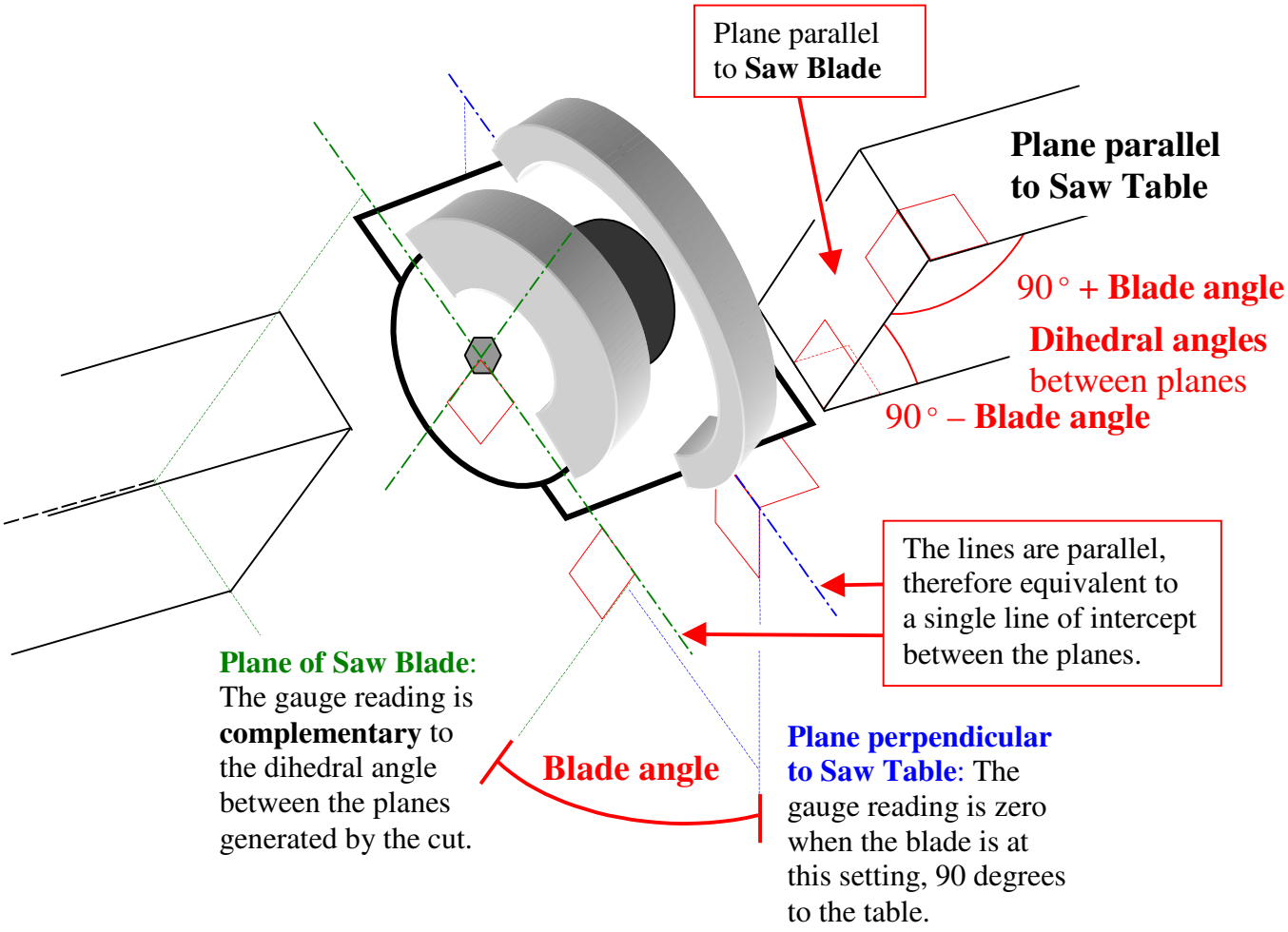
Above: Dihedral angles measured with respect to the plane created by cutting a compound angle. The timber is rectangular in cross-section.

Right: Section removed from the work; this is the standard model for analysis of the relationships in a compound angle.

SAW BLADE ANGLES

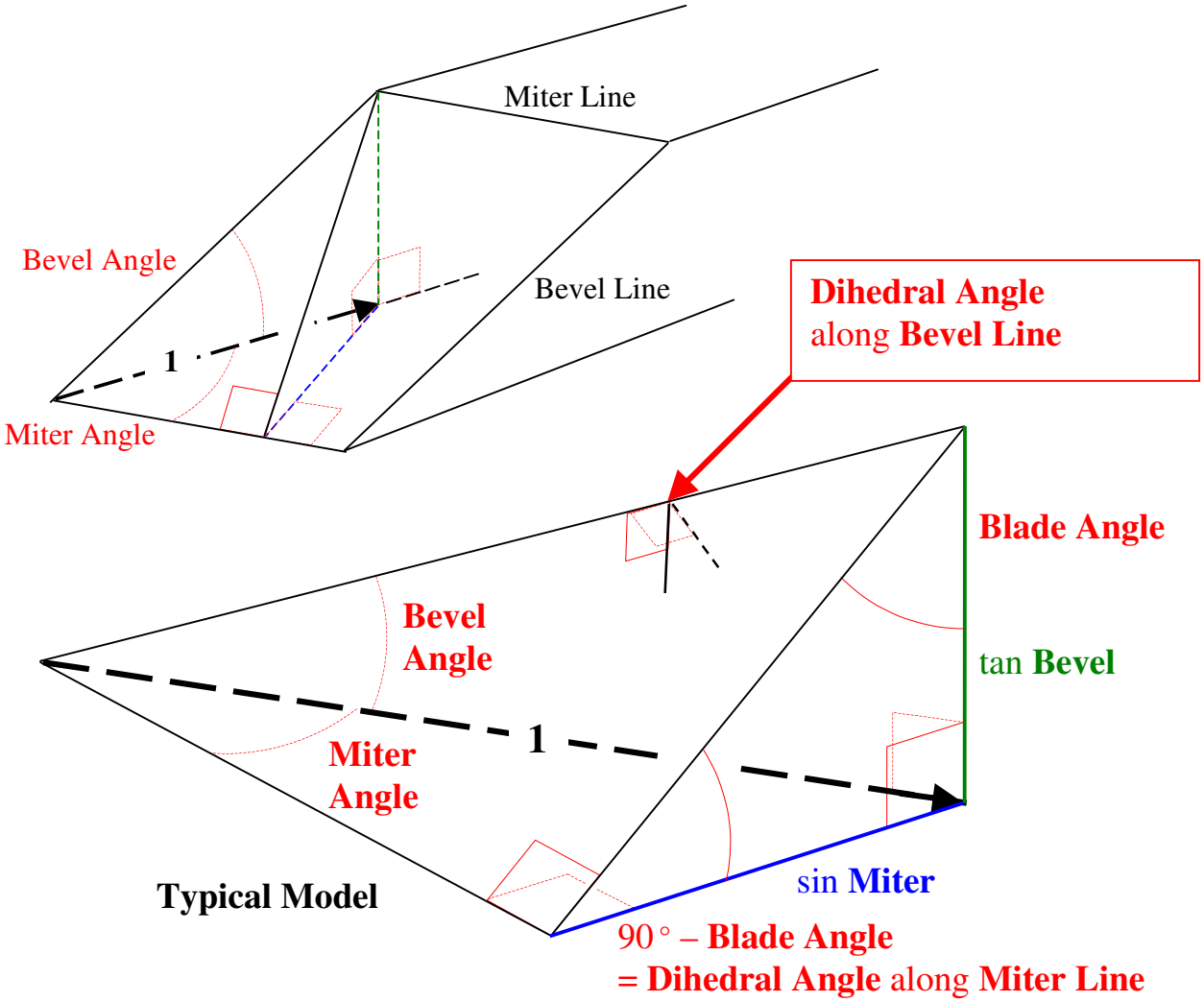
Measurement and Application of Dihedral Angles

The table and blade of a saw represent two planes, but the method of measuring the dihedral angle between them must be modified. The gauge on a saw reads zero when the angle between the table and blade is 90 degrees.



The diagram above depicts a simple cross cut at right angles to the edge of a rectangular timber. Lines parallel to the centerline of the saw blade, on planes parallel to the plane of the blade, are equivalent lines. Similarly, any lines parallel to a line perpendicular to the plane of the saw table may be treated as the same line. The lines described are all at **right angles to the line of intercept** between the two planes of interest, and meet the conditions required for the correct measurement of a dihedral angle.

SAW BLADE ANGLE FORMULA

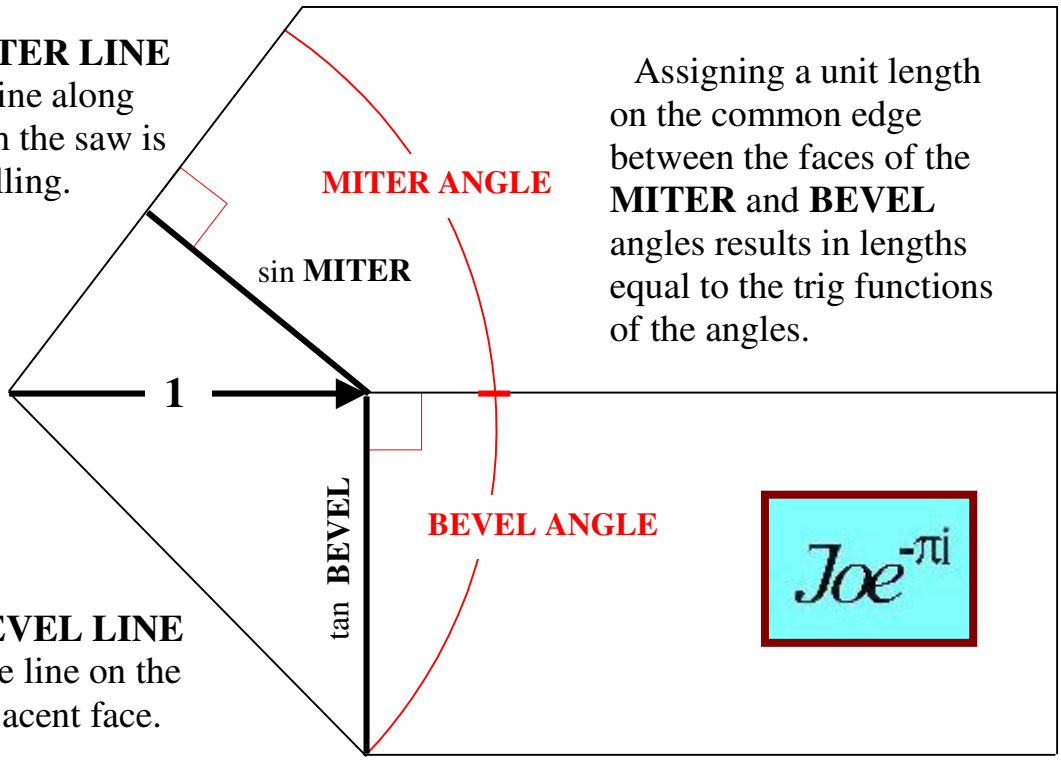


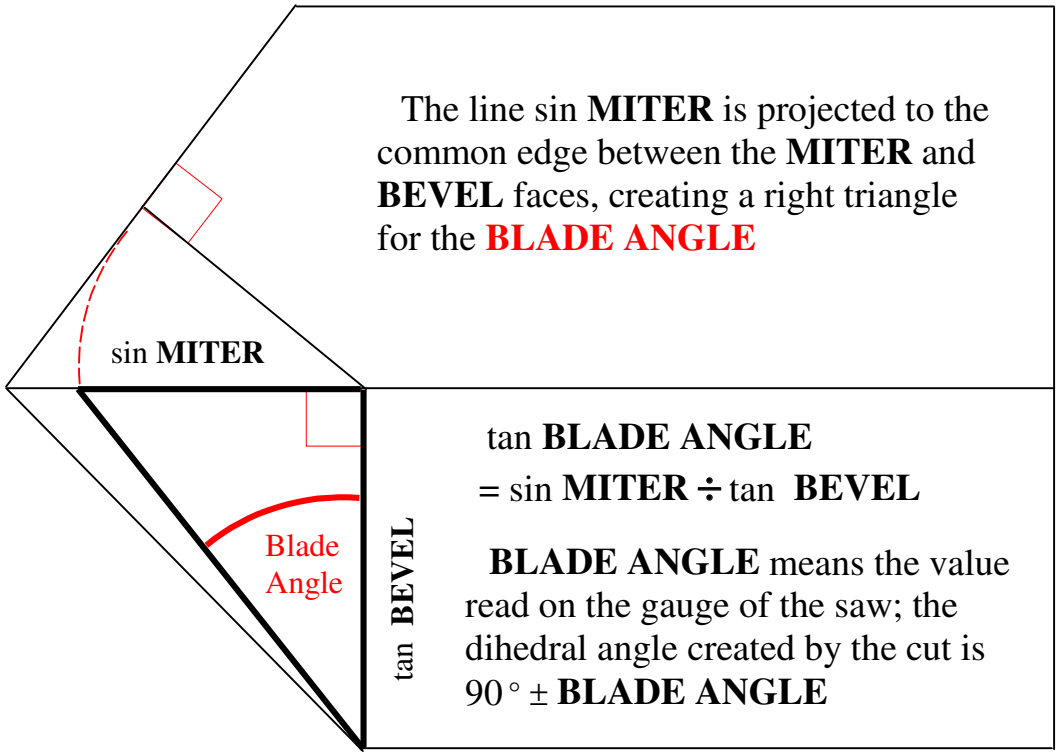
Assigning a **unit value** to the dimension shown in the diagram creates trig functions for the **Miter** and **Bevel** angles.

tan **BLADE ANGLE** = sin **MITER** ÷ tan **BEVEL**

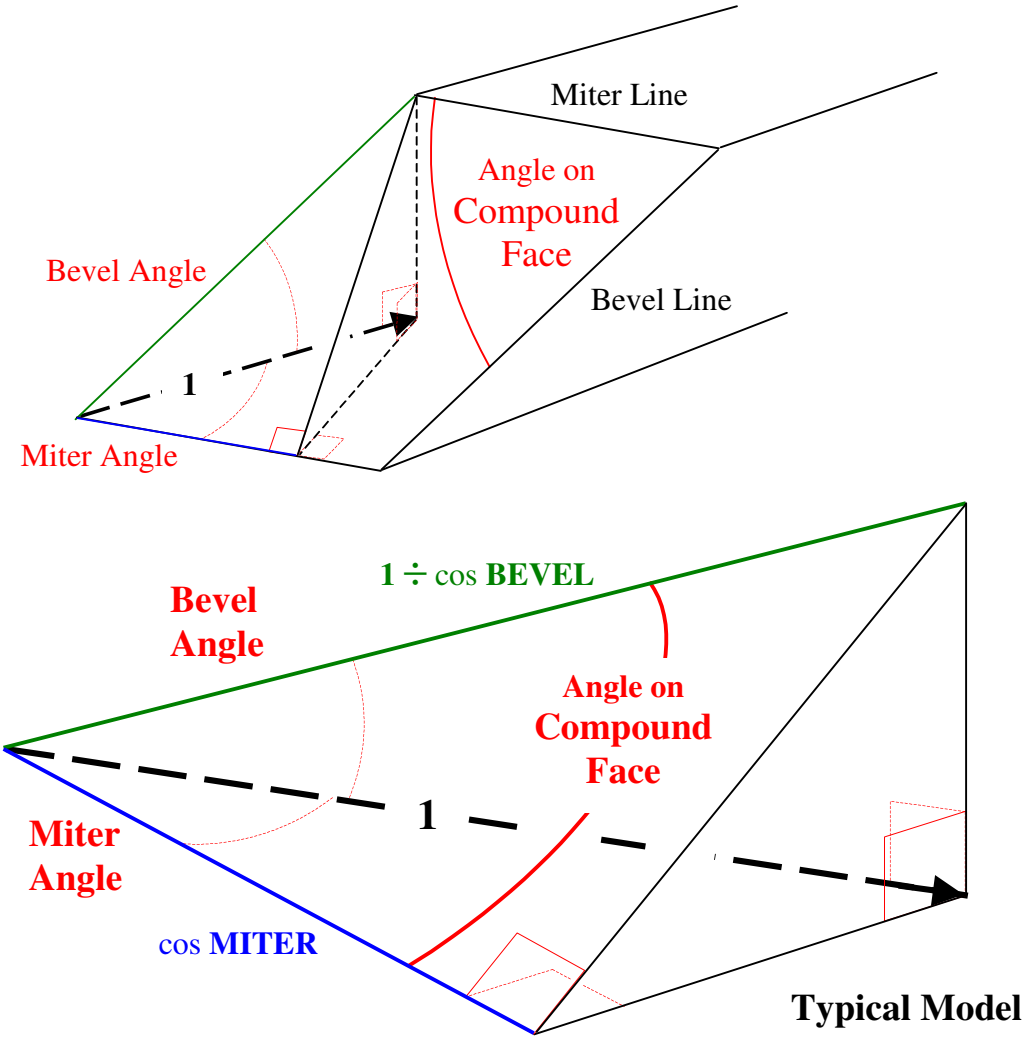
DEFINITIONS of ANGLES:
Miter Line and **Angle**: The line or angle along which the saw travels.
Bevel Line and **Angle**: The line or angle on the adjacent face of the member.
Blade Angles: The saw blade angle setting as read on the gauge; normally, a reading of zero is at 90° degrees to the saw table.
Dihedral Angle: The angle between two planes. The dihedral angle between the Compound Face created by the cut, and the face of the Miter line the blade travels on, is 90° ± **Blade Angle**.

SAW BLADE ANGLE DEVELOPED on the TIMBER





ANGLE on the COMPOUND FACE



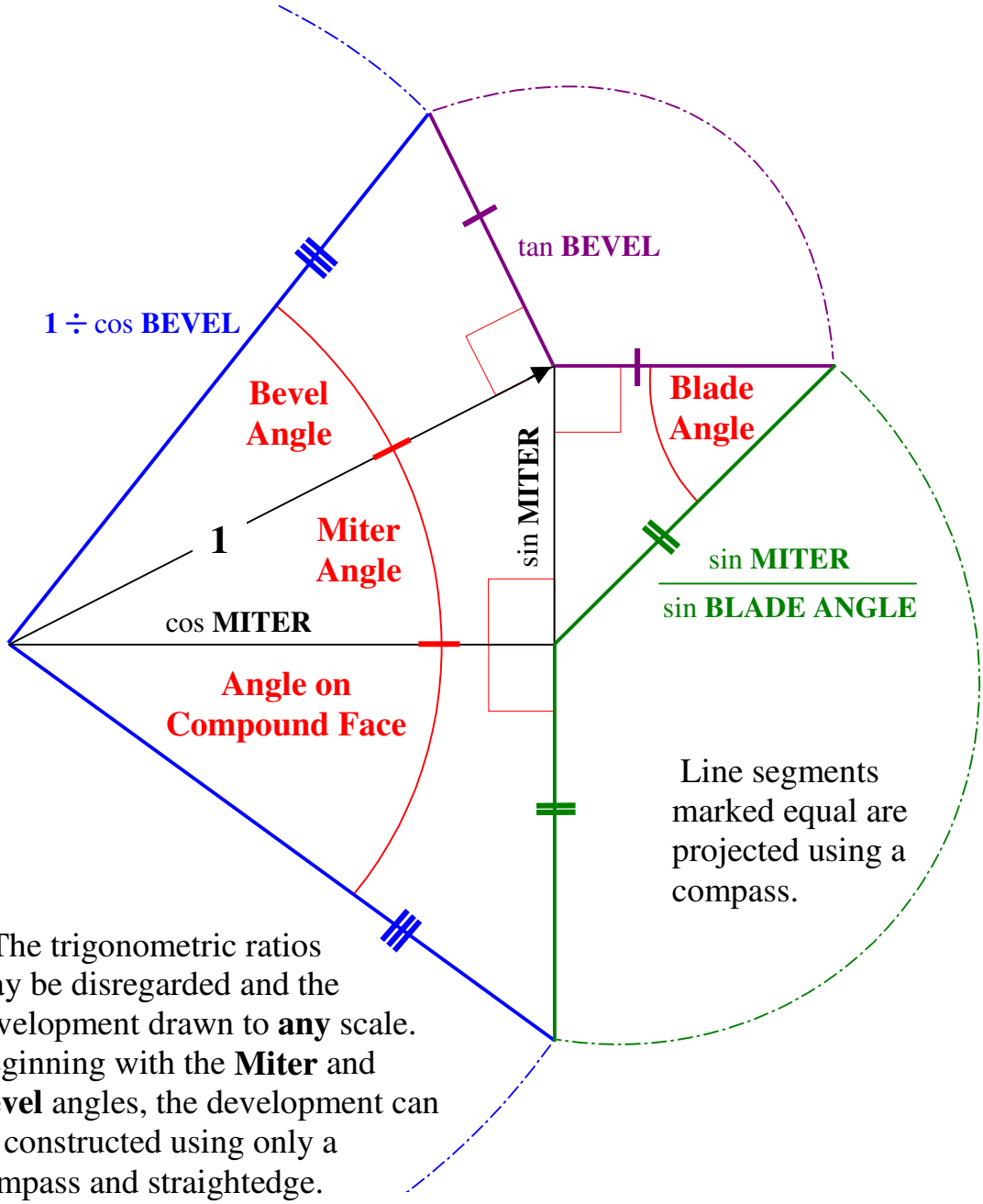
Typical Model

Cutting the compound angle on a timber rectangular in section creates a face with two pairs of supplementary angles. There is more than one formula for these angles; the easiest to remember involves the cosines of the **Miter** and **Bevel** angles.

$$\cos \textbf{ANGLE on COMPOUND FACE} = \frac{\cos \textbf{MITER}}{1 \div \cos \textbf{BEVEL}}$$

$$\cos \textbf{ANGLE on COMPOUND FACE} = \cos \textbf{MITER} \cos \textbf{BEVEL}$$

DEVELOPED MODEL of a COMPOUND ANGLE



The trigonometric ratios may be disregarded and the development drawn to **any** scale. Beginning with the **Miter** and **Bevel** angles, the development can be constructed using only a compass and straightedge.

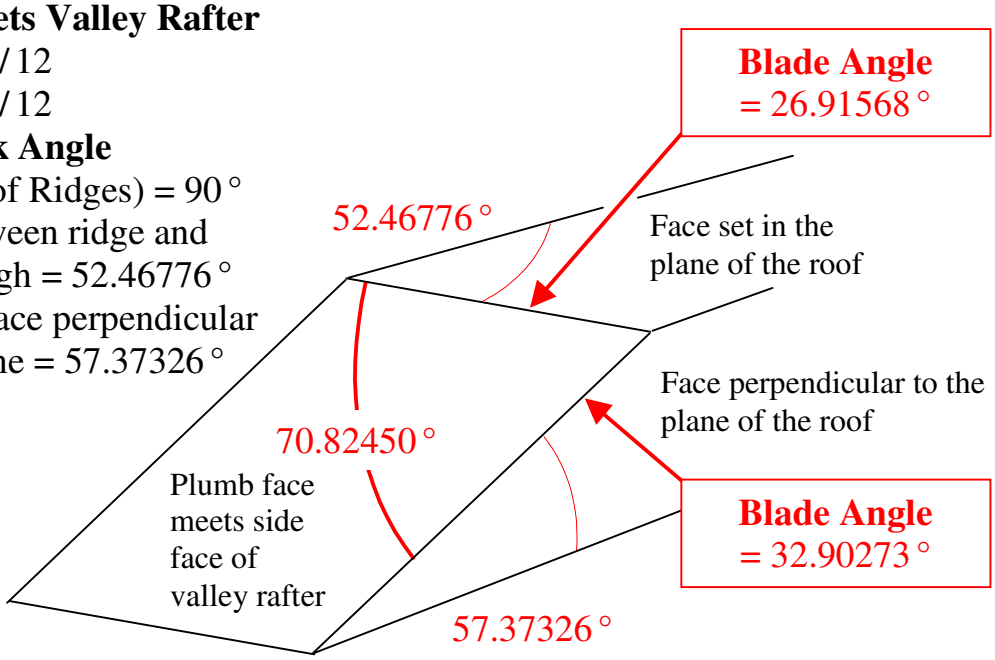
Or, the ratios in the diagram can be exploited to resolve the following formula for the **Angle on the Compound Face**:

$$\tan \textbf{ANGLE on COMPOUND FACE} = \left[\frac{\sin \textbf{MITER}}{\sin \textbf{BLADE ANGLE}} \right] \div \cos \textbf{MITER}$$

$$= \frac{\sin \textbf{MITER}}{\sin \textbf{BLADE ANGLE} \ \cos \textbf{MITER}} = \tan \textbf{MITER} \div \sin \textbf{BLADE ANGLE}$$

SAMPLE CALCULATIONS

Purlin meets Valley Rafter
Pitch = 10 / 12
Pitch = 10 / 12
Total Deck Angle
(Intercept of Ridges) = 90 °
Angle between ridge and valley trough = 52.46776 °
Angle on face perpendicular to roof plane = 57.37326 °



Let the angle between the ridge and valley trough, 52.46776 °, be the **MITER**.

tan **BLADE ANGLE =** sin **MITER** ÷ tan **BEVEL**
BLADE ANGLE = arctan (sin **MITER** ÷ tan **BEVEL**)
= arctan (sin 52.46776 ° ÷ tan 57.37326 °)
= 26.91568 ° (equal to the Backing angle)
cos **ANGLE on COMPOUND FACE** = cos **MITER** cos **BEVEL**
ANGLE on COMPOUND FACE = arccos (cos **MITER** cos **BEVEL**)
= arccos (cos 52.46776 ° cos 57.37326 °)
= 70.82450 °; this is the angle required to lay out the housing for the intercepting purlin on the side face of the valley rafter.

Which value is designated the **Miter angle** is arbitrary. Due to the depth of the work, and available blade depth at the given saw blade angle, it may be necessary to make the cut along the adjacent face.
Let the **MITER** = 57.37326 °, and **BEVEL** = 52.46776 °:

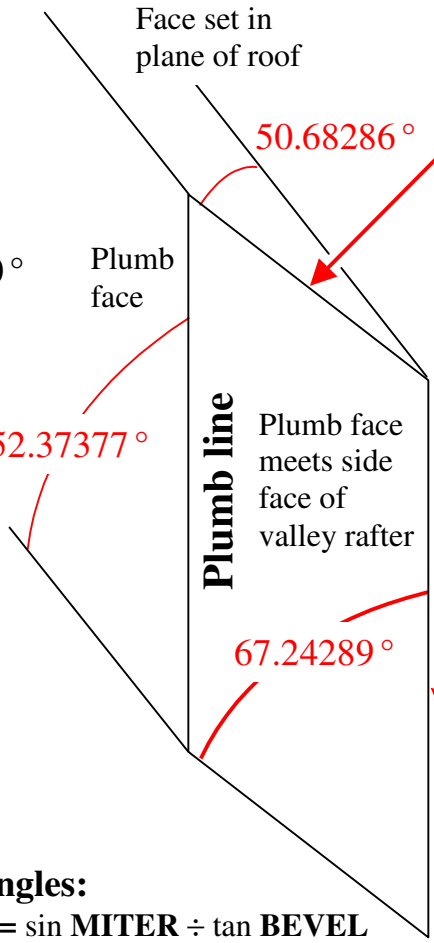
BLADE ANGLE = arctan (sin **MITER** ÷ tan **BEVEL**)
= arctan (sin 57.37326 ° ÷ tan 52.46776 °)
= 32.90273 °

Checking the math by using a different formula for the **Angle on the Compound Face**:
tan **ANGLE on COMPOUND FACE** = tan **MITER** ÷ sin **BLADE ANGLE**
ANGLE on COMPOUND FACE = arctan (tan **MITER** ÷ sin **BLADE ANGLE**)
= arctan (tan 57.37326 ° ÷ sin 32.90273 °)
= 70.82540 °

If the **Miter angle** is 52.46776 °, the **Saw Blade angle** = 26.91568 °, and:
ANGLE on COMPOUND FACE = arctan (tan **MITER** ÷ sin **BLADE ANGLE**)
= arctan (tan 52.46776 ° ÷ sin 26.91568 °)
= 70.82540 °, the same result.

SAMPLE CALCULATIONS

Jack Rafter
meets Valley Rafter
Main Pitch = 9¼ / 12
Dormer Pitch = 6 / 12
Total Deck Angle
(Intercept of Ridges) = 90 °



Blade Angle
= 30.80998 °

9¼ / 12 side values
from worksheet:
Common pitch angle
= 37.62623 °
Deck angle
= 32.96940 °
Valley pitch angle
= 22.75711 °
Angle between ridge
and valley trough
= 39.31714 °
Backing angle
= 30.80998 °

Blade Angle
= 32.96940 °

Calculating the angles using formulas, from the following known values for a **Jack Rafter** 9¼ / 12 side:
Angle for plumb line
= 90 ° – 37.62623 °
= **Miter angle**
= 52.37377 °
Deck angle
= **Blade angle**
= 32.96940 °

Solving the remaining angles:

tan **BLADE ANGLE =** sin **MITER** ÷ tan **BEVEL**

The **Bevel angle** is on the rafter face on the plane of the roof.
Transposing the terms in the equation to solve for the **Bevel angle**:

tan **BEVEL =** sin **MITER** ÷ tan **BLADE ANGLE**
BEVEL = arctan (sin **MITER** ÷ tan **BLADE ANGLE**)
= arctan (sin 52.37377 ° ÷ tan 32.96940 °)
= 50.68286 ° = 90 ° – angle between ridge and valley trough

The Valley pitch angle for the combination of given pitches is 22.75711 °.
The plumb line along the valley created by the jack rafter is 90 ° – 22.75711 ° = 67.24289 °; this is also the **Angle on the Compound Face** of the jack rafter.

cos **ANGLE on COMPOUND FACE** = cos **MITER** cos **BEVEL**
ANGLE on COMPOUND FACE = arccos (cos **MITER** cos **BEVEL**)
= arccos (cos 50.68286 ° cos 52.37377 °)
= 67.24289 °

Let the **Miter angle** be the angle on the plane of the roof.
To solve for the **Blade angle**, **Miter** = 50.68286 °, and **Bevel** = 52.37377 °:

tan **BLADE ANGLE =** sin **MITER** ÷ tan **BEVEL**
BLADE ANGLE = arctan (sin **MITER** ÷ tan **BEVEL**)
= arctan (sin 50.68286 ° ÷ tan 52.37377 °)
= 30.80998 °, the value of the Backing angle

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