A New Physical Model for Calculation of Atomic Mass

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According to the generally accepted physical model, the synthesis of the heavy elements may happen at a very high temperature in supernova explosions. In consequence of nuclear fusion, the supernova stars emit a very strong electromagnetic (EM) radiation, predominantly in the form of X-rays and gamma rays. The intensive EM radiation drastically decreases the masses of the exploding stars, directly causing mass defects of the nuclei. The general description of black body EM radiation is based on the famous Planck's radiation theory, which supposes the existence of independent quantum oscillators inside the black body. In this paper, it is supposed that in exploding supernova stars, the EM radiating oscillators can be identified with the nascent heavy elements loosing their specific yields of their own rest masses in the radiation process. The final binding energy of the nuclei is additionally determined by strong neutrino radiation, which also follows the Maxwell-Boltzmann distribution in extremely high temperature. Extending Planck's radiation law for discrete radiation energies, a very simple formula is obtained for the theoretical description of the measured neutral atomic masses. **Keywords:** atomic mass calculation, Planck's radiation law, the origin of the elements, binding energy of the nuclei, new theoretical model of the nuclear synthesis.

1. Introduction

The theoretical determination of neutral atomic masses dates back to 1935, when C.F. von Weizsäcker [1] published his famous liquid-drop model for the calculation of nuclear masses. The model provides a general overview of masses and related stability of nuclei, and assumes the nucleus behaves in a gross collective manner, similar to a charged drop of liquid. The semi-empirical mass formula based on this phenomenological model was applied successfully mainly in the earlier period of nuclear physics. From the simple drop model one can easily calculate approximately the mass of a neutral atom by adding the number Z of electron masses to the mass of $X(A,Z)$ nuclei.

However, the liquid drop model does not give answers for many important questions related to the structure and forces inside the nuclei. The long-standing goal of nuclear physics has been to understand how the structure of nuclei arises from the interactions between the nucleons. It is known that nucleons are composed of quarks, but nuclear interactions have not yet been derived successfully from the fundamental interactions between the quarks.

The standard method for the modern calculation for light nuclei is based mainly on non-relativistic quantum mechanics. In the world, many realistic phenomenological models of two- and three-nucleon interactions have been developed by fits to nucleon-nucleon (NN) scattering data and the properties mainly of ^2H, ^3H, and ^4He. The non-relativistic Hamiltonian used typically contains two-body and three-body potentials. Different types of approximation method are now available in the literature to solve the few-body problems; nevertheless, parametric fitting of experimental data still remains necessary.

The present paper does not investigate further the nuclear structure and forces, thus avoiding the difficult theoretical treatments and calculations. It focuses only on the birth of the nuclei in stars, and gives a very simple physical model for that.

It is widely known that the majority of the elements in the periodic table are synthesized in the stars. The synthesis of the heavy elements may happen only at very high temperature, in supernova star explosions. In consequence of the nuclear fusion, the supernova stars and of course, the ordinary stars, emit very strong electromagnetic (EM) radiation, predominantly in the form of gamma and X-rays. In addition, the EM radiation is combined with strong neutrino radiation, which also follows the Maxwell-Boltzmann distribution in the case of extremely high temperature. The intensive energy radiation continuously decreases the masses of the stars, directly causing the mass defects of the nascent nuclei, and at least the strong binding of nuclei. The individual nuclei represent quantized black body oscillators; their frequencies are determined by their mass numbers. From this simple physical model, one can conclude that the binding energy curve of the nuclei is in immediate connection to the Planck's radiation law in a very high temperature region. In our paper, we have fitted the Planck's radiation law to the binding energy curve of the nuclei, supposing that the radiation frequency of an arbitrary nucleon is proportional to the root-square of its mass number.

2. Extension of Planck's Radiation Law

According to the Planck's radiation law the energy density of the EM radiation in function of the radiation frequency is:

$$d\varepsilon = \frac{8\pi h}{c^3}\frac{f^3}{e^{hf/kT}-1}df \tag{1}$$

where T is the absolute temperature, c is the speed of light, k is Boltzmann's constant, h is the Planck's constant and f is the radiation frequency. The new model for explaining the origin of the elements requires discrete radiation frequencies of the stars depending on the mass numbers of the nuclei. In classical electrodynamics, the radiation energy density of a simple dipole antenna is proportional to the frequency on the fourth power:

$$\varepsilon_f = \text{constant} \times f^4 \tag{2}$$

From the analogy, the discrete energy emitted by the individual nuclei at absolute temperature T must be:

$$E_{\text{rad}}(A,Z) = \text{constant} \times \frac{f^4}{e^{hf/kT} - 1} \quad ; \quad f = f(A,Z) \tag{3}$$

The binding energy of the nuclei $X(A,Z)$ is equal to the negative value of the emitted energy. One can suppose that Eq. (3) is a natural generalisation of the Planck's radiation law for discrete radiation frequencies.

The most important task was to determine the mathematical relation between the radiation frequency and the arbitrary nucleon signed $X(A,Z)$. Our original goal was to calculate the neutral atomic mass values; therefore, it was supposed that the radiation frequency mainly depends on its mass; *i.e.*. on the mass number A. It is also clear that the Z dependence in the frequency expression may be very small, and it can take account later as a small correction. Generally, the connection between the mass and its frequency is very simple:

$$E_0 = mc^2 = ma^2\omega^2 \quad , \tag{4}$$

according to the Special Relativity Theory (SRT). From this relation, it is obvious that the square frequency that can be associated with the atom is very nearly proportional to the mass number of the atom:

$$f^2(A) = \text{constant} \times A \quad . \tag{5}$$

3. New Mass-Formula for Neutral Atoms

Neglecting the Z-dependence of the neutral atoms, the mass calculation is based on the next simple expression:

$$M(A) = AM_0 - E_{\text{rad}}(A)/c^2 \quad , \tag{6}$$

where M_0 is a phenomenological constant and $E_{\text{rad}}(A)$ is the energy emitted from the star related to the nucleon with mass number A. The detailed form of (6) gives:

$$M(A) = AM_0 - c_1 \frac{f^4}{e^{hf/kT} - 1} \quad ; \tag{7a}$$

where it was supposed by (5) that

$$f = f(A) = c_2\sqrt{A} \quad . \tag{7b}$$

Replacing new variables for the unknown constants c_1 and c_2, we have:

$$M(A) = M_0\left[A - \tfrac{1}{2}\lambda F^4 \big/ (\eta^F - 1)\right]; \quad F = \sqrt{A} \tag{8}$$

where λ, η and M_0 are the new fitting parameters for the experimentally determined atomic mass values. Here a *new variable* F is introduced *proportional to the radiation frequency*.

4. Numerical Results

In the numerical procedure *we have observed*, that the parameter λ precisely equal to an important mass ratio:

$$\lambda = m / M = \text{electron mass} / \text{proton mass} \quad , \tag{9}$$

and therefore the mass formula will be:

$$M(A) = M_0\left[A - \tfrac{1}{2} mF^4 \big/ M(\eta^F - 1)\right] \quad . \tag{10}$$

For the variable F defined in (8), we have found a better expression:

$$F = \sqrt{A - 2} \tag{11}$$

which means that the binding energy of the deuteron is very small, its radiation frequency is approximately zero compared to heavier nuclei.

The Eqs. (10) and (11) yield excellent agreement with the average trend of the measured masses of all stable atoms *except those of very small* A. For this reason we have tried to decrease the values F^4 for small atomic numbers. After some investigation, we have obtained a very successful expression:

$$F^4 = A(A - 6) \quad . \tag{12}$$

Finally, the new atomic mass formula can be written:

$$M(A) = AM_0\left[1 - \tfrac{1}{2} m(A-6) \big/ M(\eta^{\sqrt{A-2}} - 1)\right] \quad , \quad (A \geq 3) \tag{13}$$

which involves only two fits: M_0 and η.

The new mass formula represented by Eq. (13) was fitted to nearly 2000 measured neutral atomic masses obtained from the publication of G. Audi and A.H. Wapstra [2]. The results of the fitting procedure are:

$$M_0 = 1.003303846 \text{ a.u.}; \quad \eta = 1.220126766 \tag{14}$$

The accuracy of the new formula was determined by the relative standard deviation:

$$\sigma = \sqrt{\frac{1}{N}\sum \frac{(M_{\text{calculated}} - M_{\text{measured}})^2}{M_{\text{measured}}^2}} = 3.216 \times 10^{-4} \tag{15}$$

which is an excellent result. Fig. 1 shows the relative errors of the fitted atomic masses.

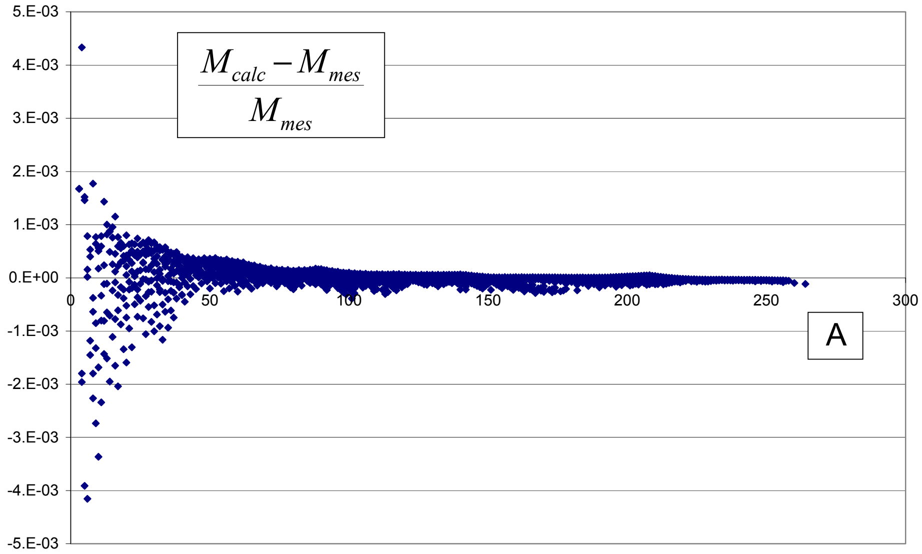

Figure 1. Relative deviations between calculated and measured atomic masses

5. Conclusions

Based on our new successful atomic mass formula, we have concluded that at extreme high temperature nuclear synthesis can be physically described *exclusively following* the Planck's radiation law. In Planck's model it is supposed that *black body oscillators* are independent of each other, and have a Maxwell-Boltzmann energy distribution. Already in earlier nuclear physics, there was some experience showing that all the nuclei inside the atom are weakly bound. This experimental fact was also proved theoretically here in the present work.

The accuracy of the here-introduced two-parameter formula is comparable with the accuracy of the *five-parameter* liquid drop model established by von Weizsäcker. Nevertheless, the new calculation model has a serious chance to improve its accuracy by taking account additionally the Z -dependence of the atomic masses and/or additional experimentally known parameters (nucleon-spin, parity, *etc*.).

Acknowledgement

The author would like to acknowledge the Hungarian Research Centre of Fundamental Physics for support of the realization of the present work. The author is also very grateful to colleagues of RFP for providing interest and encouragement in developing of the new model for atomic mass calculation.

References

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